Tchebichef Moment Transform for Colour Image Dithering

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***Abstract*-Currently, small mobile devices may not have the capability to display high fidelity true colour images. It is expected to have limited capability in computational power and storage. However, a small mobile device is expected to perform efficiently in displaying images. A dithering technique is called for here in order to improve the perceptual visual quality of the limited bit depth images. Typically, a true colour image is reduced down to fewer bits per pixel. In order to efficiently dither a colour image on small mobile devices, this paper proposes 2×2 Tchebichef Moment Transform (TMT) approach. Earlier, a 2×2 Discrete Wavelet Transform (DWT) has been proposed for better image quality on dithering. The 2×2 TMT has been chosen since it performs better than the 2 × 2 DWT. TMT incorporates simplified mathematical framework technique using matrices. The result shows that 2×2 TMT gives perceptually better quality on colour image dithering in significantly efficient fashion.**

I. INTRODUCTION

A true colour image is represented by 24-bit colours for each pixel on the screen display. The full colour image display is capable of generating up to 16 million different colours. However, a higher computing power is needed to support such a display on high fidelity images. At the same time, many mobile devices have a limited number of colours to display a true colour image. Mobile devices are only expected to be equipped with low computing power and storage. A mobile image application has to do with the limited number of bit allocated for each colour per pixel. Alternatively, an image dithering is a useful technique to reduce the colour depth in the image displayed. The aim of dithering is to distribute errors among pixels to exploit visual perception colour images displayed with a limited colour option. Direct reduction of the bit depth will result in image staggering. The uses of dithering technique will produce perceptually smoothen the image displayed through using limited colours.

Previously, an image dithering using 2×2 TMT has been applied on grayscale images [1]. In this paper, image dithering based on 2×2 Tchbichef moment is utilized on colour images dithering. A 2×2 pixel block is chosen here instead the popular standard 8x8 sub block image to produce minimum error reconstruction. In the previous research, image dithering for gray scale image based on 2×2 TMT performed better than 2 × 2 DWT [1]. However, DWT perform higher computations and it requires a special wavelet filter to analyze and reconstruct the signal. In order to overcome the complexity of DWT, this paper proposes 2×2 Tchebichef moments for efficiency and simplicity to dither colour image. Results also show that TMT requires lower computation to dither colour image than DWT. This paper will not only inspect the image quality visually but also evaluate the difference between the original image and its reconstructed image.

TMT does not involve any numerical approximation unlike other popular continue transforms. The Tchebichef moment consists of rational numbers only. In addition, Tchebichef moment requires a simple evaluation of algebraic expression only. Without going into complex field, TMT has been widely used in image and audio processing. For examples, they are used in image analysis [2][3], texture segmentation, multispectral texture, template matching, pose estimation, pattern recognition, image reconstruction [4], image projection [5], image compression [6]-[9] and speech recognition [10]-[13].

The organization of this paper is as follows. The next section will give a brief description on DWT and the proposed TMT on colour image dithering. Section III presents the experiment result of colour image dithering based on Floyd Steinberg, 2×2 DWT and 2×2 TMT. Next, the comparison of experiment results is discussed in Section IV. Lastly, Section V will conclude this paper.

II. DISCRETE WAVELET TRANSFORM AND TCHEBICHEF MOMENT TRANSFORM

A. Discrete Wavelet Transform

The wavelet transform is computed separately for different segment on the time domain signals at different frequencies. DWT uses multi resolution filter banks and special wavelet filters for the analysis and reconstruction of signals. Filtering the image with 2-D DWT increases the phase distortion. Most DWT implementations use separable filtering with real coefficient filters associated with real wavelets resulting in real valued approximations and details. The decomposition of DWT in the image is shown in Figure 1.

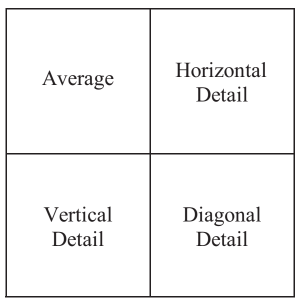

Figure 1. Wavelet Decomposition Structure.

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In two dimensions, a scaling function $\varphi(x,y)$ and three wavelets $\Psi^H(x,y)$, $\Psi^V(x,y)$ and $\Psi^D(x,y)$ are necessary. The $\Psi^H(x,y)$ measures variations along columns (horizontal), $\Psi^V(x,y)$ responds to variation along rows (vertical) and $\Psi^D(x,y)$ corresponds to variations along diagonals (diagonal). In 2D-DWT, the scaled and translated basis functions are defined by [14]:

$$\Phi_{j,m,n}(x,y) = 2^{\frac{j}{2}}\varphi(2^j x - m, 2^j y - n) \quad (1)$$

$$\Psi^{i}{}_{j,m,n}(x,y) = 2^{\frac{j}{2}}\Psi^i(2^j x - m, 2^j y - n), i = \{H,V,D\} \quad (2)$$

where index i identifies the directional wavelet in terms of value of H (Horizontal), V (Vertical) and D (Diagonal). The DWT of function $f(x,y)$ of size 2×2 is given as follows:

$$W_\varphi(j_0,m,n) = \frac{1}{\sqrt{MN}}\sum_{x=0}^{1}\sum_{y=0}^{1} f(x,y)\varphi_{j_0,m.n}(x,y) \quad (3)$$

$$W_\Psi^i(j,m,n) = \frac{1}{\sqrt{MN}}\sum_{x=0}^{1}\sum_{y=0}^{1} f(x,y)\Psi^{i}{}_{j,m.n}(x,y) \quad (4)$$

where $i = \{H,V,D\}$, j_0 is the starting scale, the $w_j(j_0,m,n)$ coefficients define the approximation of $f(x,y)$, $W_\Psi^i(j,m,n)$ coefficients represent the horizontal, vertical and diagonal details for scales $j \geq j_0$. Here $j_0 = 0$ and select $N + M = 2^j$ so that $j = 0,1$ and $m,n = 0,1$. Then the inverse of the DWT is given as follows:

$$f(x,y) = \frac{1}{\sqrt{MN}}\sum_m\sum_n W_\varphi(j_0,m,n)\varphi_{j_0,m.n}(x,y) +$$

$$\frac{1}{\sqrt{MN}}\sum_{i=H,V}\sum_{D\,j=j_0}^{\infty}\sum_m\sum_n W_\Psi^i(j_0,m,n)\Psi^{i}{}_{j_0,m.n}(x,y) \quad (5)$$

DWT has shown to perform well in image processing applications. It has been used in image compression, image watermarking, texture analysis and image dithering [15]. Image dithering based on DWT suggested transform the full image down to 2×2 pixels.

B. Tchebichef Moment Transform

Tchebichef moment transform is a transform method based on discrete orthogonal Tchebichef polynomials [2], which has energy compactness properties for both graphical and natural images. For a given set $\{t_n(x)\}$ of input a value (image intensity values) of size $N = 2$, the forward discrete orthogonal Tchebichef Moments of order $m + n$ is given as follows [2]:

$$T_{mn} = \frac{1}{\rho(m,N)\rho(n,N)}\sum_{x=0}^{1}\sum_{y=0}^{1} t_m(x)t_n(y)f(x,y) \quad (6)$$

for $m = 0,1$ and $n = 0,1$

where $f(x,y)$ denotes the intensity value at the pixel position (x,y) in the image. The $t_n(x)$ are defined using the following recursive relation:

$$t_0(x) = 1, \quad (7)$$

$$t_1(x) = \frac{2x + 1 - N}{N}, \quad (8)$$

The set $\{t_n(x)\}$ has a squared-norm given by

$$\rho(n,N) = \sum_{i=0}^{1}\{t_i(x)\}^2$$

$$= \frac{N.\left(1 - \frac{1^2}{N^2}\right).\left(1 - \frac{2^2}{N^2}\right)}{2n + 1} \quad (9)$$

The description of squared-norm $\rho(.)$ and the properties of orthogonal Tchebichef polynomials are given in [2]. The process of image reconstruction from its moments, the inverse moment Tchebichef moments are given as follows:

$$f(x,y) = \sum_{m=0}^{1}\sum_{n=0}^{1} T_{mn}t_m(x)t_n(y) \quad (10)$$

for $m = 0,1$ and $n = 0,1$

where m denotes the maximum order of moments used and $\tilde{f}(x,y)$ the reconstructed intensity distribution. Tchebichef moment transform has its own advantage in image processing which has not been fully explored. The Discrete orthogonal Tchebichef polynomial domain consists of real rational numbers unlike the continuous orthogonal transform. Discrete orthogonal Tchebichef moment is capable of performing image reconstruction exactly without any numerical errors [4].

III. Experiments

This section presents the experimental methods and results of colour image dithering. In this experiment, Floyd Steinberg method, DWT and TMT shall be used on image dithering. This experiment used 80 sample images containing 40 graphic images and 40 natural images. Originally images were 24-bit RGB image with the of size 512×512 pixels and has reduced the bit depth colour image to 4-bit using the popular clustering technique, which is K-means method. The samples of 40 graphic images and 40 natural images of size 512×512 pixels were analyzed and evaluated on image dithering. A sample true colour of natural and graphic images with size 512×512 pixels is shown in Figure 2.

Figure 2. Sample of 24-bit RGB of natural image (left) and graphic image (right) with 512×512 pixels (scale to 50%).

Next, original true colour images are reduced down to 16 colours or 4-bit using K-Means algorithm.

A. K-Means Algorithm

The clustering approaches can be categorized into partition and hierarchical clustering algorithms. The partition clustering algorithms such as K-means clustering is a popular technique for clustering colour image processing by partitions data set into k sets or a number of distinct groups. Typically K-mean begins with k arbitrary centers, it is chosen uniformly at random from the data points. Each point is then assigned to the nearest center and each center is recomputed as the center of mass of all points assigned to it. These two steps (assignment and center calculation) are repeated until the process stabilizes. K-Means algorithm attempts to find the cluster centre $s_1, \dots, s_j$, that sum of squared distance of each data point x_i to its nearest cluster centre s_j is minimized. The membership for each data point belongs to nearest centre depends on minimum distance. This membership is determined as follows [16]:

$$M(x,s) = \sum_{i=1}^{N} min_{j\in(1\dots k)} \|x_i - s_j\|^2, \tag{11}$$

where N refers to numbers of data point, x_i is each data point, s_j is cluster centre and k is the number of clusters the data will be partitioned. The outputs of natural and graphic images with 4-bit of size 512×512 pixels using K-Means algorithm are shown in Figure 3.

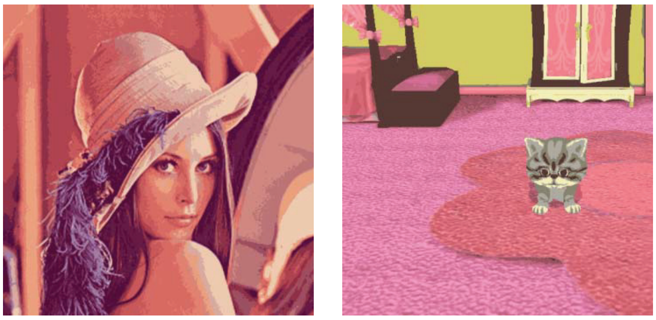

Figure 3. 4-bit RGB of natural image (left) and graphic image (right) with 512×512 pixels (scale to 50%).

The experimental result of reduced colour using K-Mean as presented in Figure 3 shows that the image output has much lower quality than original 24-bit RGB image. In the experiment, the colour image was clustered into 16 clusters. The number of clusters is determined to classify number of colour images. Having large number of pixels on true colour images, K-Means may be computationally faster than hierarchical clustering. This palette is representing the pixel colours in the sample above. The palette tables specification of the 4-bit colours on natural image and graphic image of size 512×512 pixels are presented in Table I and Table II respectively.

TABLE I
PALETTE COLOUR FOR SAMPLE NATURAL IMAGE OF SIZE 512×512 PIXELS

Indexed	R	G	B	RGB	Indexed	R	G	B	RGB
1	116	52	99		9	163	126	163	
2	200	89	91		10	158	80	98	
3	234	159	127		11	208	136	128	
4	236	201	175		12	84	17	60	
5	226	130	112		13	217	107	101	
6	143	49	71		14	138	85	134	
7	175	67	79		15	218	171	160	
8	188	115	118		16	103	30	68	

TABLE II
PALETTE COLOUR FOR SAMPLE GRAPHIC IMAGE OF SIZE 512×512 PIXELS

Indexed	R	G	B	RGB	Indexed	R	G	B	RGB
1	239	156	192		9	224	121	132	
2	216	107	119		10	190	104	127	
3	202	93	104		11	212	205	99	
4	212	111	151		12	222	126	165	
5	245	237	185		13	244	128	140	
6	230	139	178		14	137	132	111	
7	155	83	95		15	86	57	52	
8	174	169	161		16	242	152	159	

Next, the 4-bit RGB image with 16 colours is divided into 2×2 pixel blocks of pixels. Each block is transformed from the pixel domain to the moment coefficient by 2×2 TMT. The weight filter table is used to adjust the image output. Next, the error diffusion for TMT is proposed to distribute errors among its pixels. 2×2 TMT is implemented to achieve better performance on image dithering. The visualization of TMT colour image dithering scheme is presented in Figure 4.

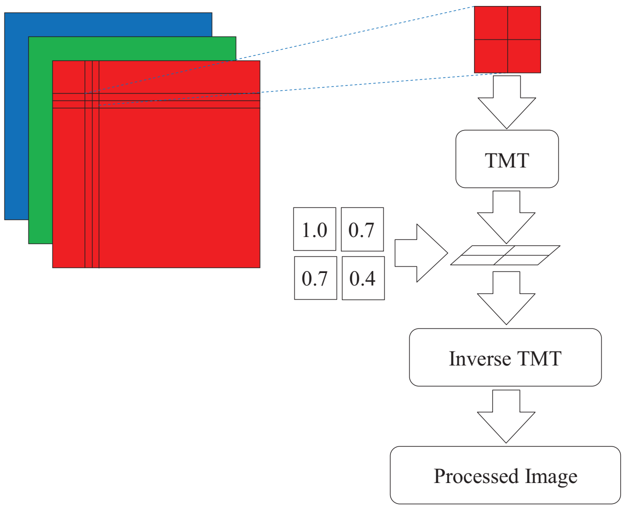

Figure 4. Visualization of Image Dithering Application based on TMT.

B. *Sub Block Image*

During the image dithering, 2×2 blocks of pixels are processed from left to right and from top to bottom. In this experiment, 2×2 sub block image becomes useful due to its size advantage in the image dithering. A 2×2 sub block image becomes popular and efficient in image reconstruction [1]. Next, each block of the image is computed with 2×2 orthogonal Tchebichef polynomials independently.

C. *Moment Coefficient*

This section provides a compact representation of the moment coefficients and the inverse moment coefficients. The block size N is taken to be 2 and they are compared to the image dithering with $N = 2$ using DWT. Based on discrete orthogonal moments as defines in equation (7)-(9), a kernel matrix $K_{(2\times2)}$ is given as follows:

$$K = \begin{bmatrix} t_0(0) & t_1(0) \\ t_0(1) & t_1(1) \end{bmatrix} \tag{12}$$

The image block matrix by $F_{(2\times2)}$ with $f(x,y)$ denotes the intensity values of the image pixels:

$$F = \begin{bmatrix} f(0,0) & f(0,1) \\ f(1,0) & f(1,1) \end{bmatrix} \tag{13}$$

The matrix $T_{(2\times2)}$ of moments is defined based on (6) above as follows:

$$T = K^T F K \tag{14}$$

This process is repeated for every block in the original image to generate the coefficients of discrete orthogonal Tchebichef Moments. The inverse moments relation used to reconstruct the image block from the above moments is given as follow:

$$G = KTK^T, \tag{15}$$

where $G_{(2\times2)}$ denotes the matrix image of the reconstructed intensity values. This process is repeated for every block on the coefficients Tchebichef Moments. In order to adjust the contrast of the image, the filtering process is applied on TMT coefficients. The filtering process is given as follows:

$$c_{(2\times2)} = P\, c_{(2\times2)}, \tag{16}$$

Where $c_{(2\times2)}$ is the TMT coefficient and P is the weight on the filtering process. The weight table is given as follows:

$$P = \begin{bmatrix} 1.0 & 0.7 \\ 0.7 & 0.4 \end{bmatrix} \tag{17}$$

The sample weight above is applied for filtering process on colour image dithering based on 2×2 TMT and 2×2 DWT respectively.

D. Error Diffusion

There are many various techniques to reduce of the bit depth colour images. The error diffusion technique is proposed by Floyd Steinberg [17] which has become a standard in half toning. The Floyd Steinberg filter is one of the popular methods based on error dispersion to distribute the error among the neighbouring pixels. The error is being dispersed among the pixels to the right and below the current pixel. The same order is also applies for TMT image dithering technique. The error diffusion Floyd Steinberg method is presented in Figure 5.

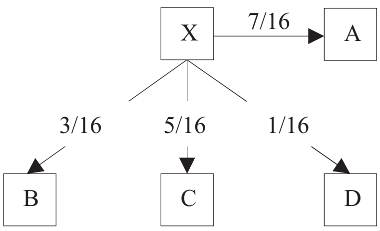

Figure 5. The Floyd Steinberg error diffusion filter.

Where X represents the current pixel and A, B, C and D represent the neighbouring pixels that receive 7/16, 3/16, 5/16 and 1/16 of the error respectively. The nearest intensity scale of current pixel is separate into 16 options. The image dithering based on DWT will apply the same error diffusion from Floyd Steinberg method to dither colour image. In order to apply Tchebichef moment, this paper proposes new scheme error diffusion in image dithering based on TMT. The error is dispersed among pixels to the right and below the current pixel according to the pattern as presented in Figure 6.

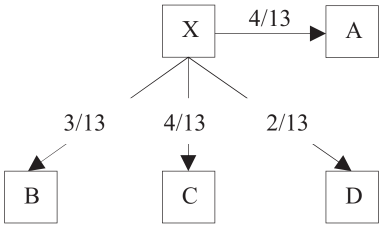

Figure 6. The proposed error diffusion filter.

The effect of error diffusion is distributed among quantize of a pixel to neighboring pixels such that the neighboring pixels are biased in the reverse direction.

E. Image Quality Measurement

The image reconstruction error is typically calculated by obtaining the differences between the dithered image $g(i,j,k)$ from the original image $f(i,j,k)$ as follows:

$$E(S) = \frac{1}{3MN} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} \sum_{k=0}^{2} |g(i,j,k) - f(i,j,k)| \tag{18}$$

where the image size is $M \times N$ and the third index refers to the RGB colours. Another convenient measurement is the Means Squared Error (MSE), it is defined as follows:

$$MSE = \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} \sum_{k=0}^{2} \| g(i,j,k) - f(i,j,k) \|^2 \tag{19}$$

The next measurement is Peak Signal to Noise Ratio (PSNR). The PSNR is defined as follows:

$$PSNR = 10 \log_{10} \left(\frac{{Max_i}^2}{\sqrt{MSE}} \right) \tag{20}$$

where Max_i is the maximum possible pixel value of the image. Some other measurement, Average Difference (AD) and Maximum Difference (MD) are also calculated for comparison. The AD and MD formulas are given as follows:

$$AD = \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} \sum_{k=0}^{2} \| g(i,j,k) \| \tag{21}$$

$$\text{MD=max(max(max(}|g(i,j,k)|\text{)))} \tag{22}$$

The average error score of colour image dithering based on Floyd Steinberg, 2×2 DWT and 2×2 TMT for 40 graphic images and 40 natural images of size 512×512 pixels are shown in Table III and Table IV.

TABLE III

AVERAGE ERROR SCORE AMONG FLOYD STEINBERG, 2×2 DWT AND 2×2 TMT FOR 40 GRAPHIC IMAGES OF SIZE 512×512 PIXELS

Evaluation Measurement	Floyd Steinberg	2×2 DWT	2×2 TMT
Full Error	8.0995	8.4331	8.4331
MSE	148.6572	165.9775	165.9814
PSNR	27.2867	26.6695	26.6695
AD	-0.0054	-0.0144	-0.0143
MD	100.8250	106.2750	105.4250

TABLE IV
AVERAGE ERROR SCORE AMONG FLOYD STEINBERG, 2×X2 DWT AND 2×2 TMT FOR 40 NATURAL IMAGES OF SIZE 512×512 PIXELS

Evaluation Measurement	Floyd Steinberg	2×2 DWT	2×2 TMT
Full Error	8.0524	7.9546	7.9549
MSE	120.8660	120.3355	120.3595
PSNR	27.7424	27.8003	27.7994
AD	-0.0045	-0.0194	-0.0194
MD	79.6250	88.3250	88.2250

In order to observe the effectiveness on visual image output, the image of a sample natural and a graphic image are zoomed in up to 400%. Here, the Lena image is used as a sample natural image to observe the visual quality perceptually. The comparison between image output of 4-bit RGB of Lena image and colour image dithering based on Floyd Steinberg on zoomed in image 400% are given in Figure 8. The visual image coming out of image dithering using 2×2 DWT and 2×2 TMT are shown on the left and right of Figure 9.

Figure 7. 8-bit RGB of Lena image with 512×512 pixels scale 25% (left) and zoomed in to 400% (right).

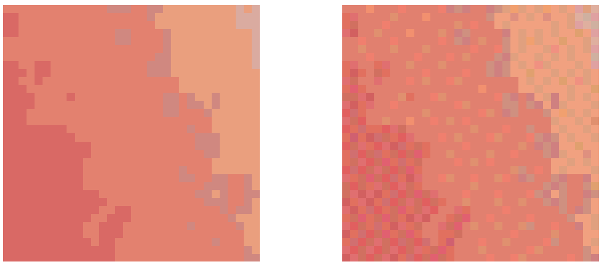

Figure 8. The 4-bit RGB of Lena image (left) and Floyd Steinberg (right) zoomed in to 400%.

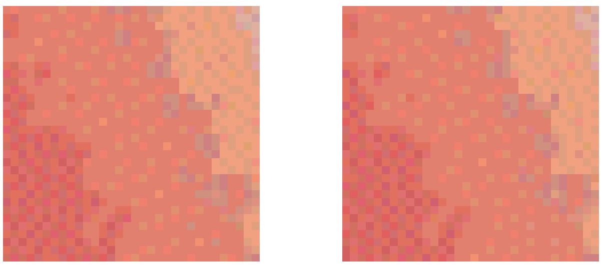

Figure 9. The visual outputs of dithered image from 2×2 DWT (left) and 2×2 TMT (right) on zoomed in to 400%.

Next, a sample graphic image is shown on the left of Figure 10. The image output of 4-bit RGB of graphic image and colour image dithering based on Floyd Steinberg are given in Figure 11. The visual image outputs of image dithering using 2×2 DWT and 2×2 TMT for graphic image are shown in Figure 12.

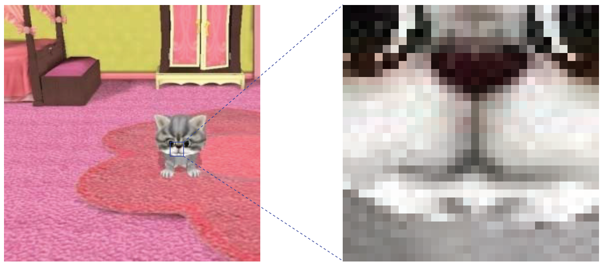

Figure 10. 8-bit RGB of graphic image with 512×512 pixels scale 25% (left) and zoomed in to 400% (right).

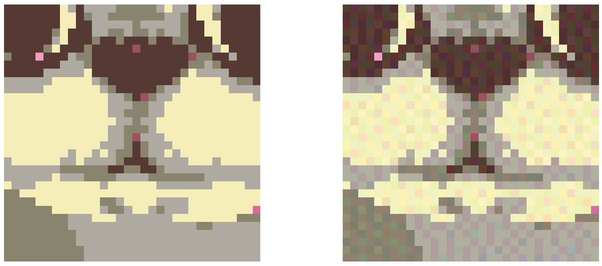

Figure 11. The 4-bit RGB of Lena image (left) and Floyd Steinberg (right) zoomed in to 400%.

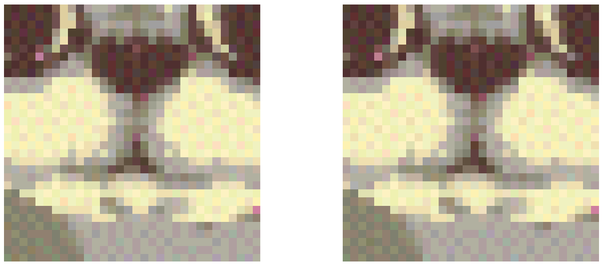

Figure 12. The visual outputs of dithered image from 2×2 DWT (left) and 2×2 TMT (right) on zoomed in to 400%.

The average time taken from colour image dithering on 40 graphic images and 40 natural images of size 512×512 pixels is shown in Table V.

TABLE V
AVERAGE TIME TAKEN COMPARISON AMONG FLOYD STEINBERG, 2×2 DWT AND 2×2 TMT FOR 40 GRAPHIC AND 40 NATURAL IMAGES

Time Taken	Floyd Steinberg	2x2 DWT	2x2 TMT
Graphic images	9.7830 sec	406.0426 sec	14.8553 sec
Natural images	11.1626 sec	344.9118 sec	15.0227 sec

IV. DISCUSSION

The experimental results of colour image dithering for 4-bit RGB image using Floyd Steinberg, 2×2 DWT and 2×2 TMT have been done. This experiment used 80 images containing 40 graphic images and 40 natural images. Image dithering based on 2×2 TMT produces more efficient computation to dither colour image than 2×2 DWT. In the previous research, image dithering based on 2×2 DWT [15] produce better visual quality on the output image than Floyd Steinberg method. In addition, DWT consumes significantly longer time to dither colour images. In general, DWT requires a lot of computation and special wavelet filter to analyze and reconstruct large images.

And image dithering based on Floyd Steinberg method, 2×2 DWT and 2×2 TMT on the 4-bit RGB image may be observed as presented in Figure 8, Figure 9, Figure 11 and

Figure 12. A dithered image output from 2×2 DWT for natural image as shown on the left of Figure 9 show that DWT produce visually better quality image than the dithered image output from Floyd Steinberg as shown on the right of Figure 8. Referring to Figure 12, a dithered image output based on 2×2 DWT for graphic image produce perceptually more natural image than a dithered image output from Floyd Steinberg as presented on the right of Figure 11 and it is closer toward the original image. Otherwise, the dithered image output based on 2×2 TMT produce visually similar quality output image than a dithered image output from 2×2 DWT. TMT dithering consistently produces perceptually similar quality output to DWT dithering at the expense faster speed.

According to the experiment results as presented in Table IV, the average error score of a dithering image output based on 2×2 TMT significantly produces better quality image measurement score than a dithered image output from Floyd Steinberg on 40 natural images. The results show that TMT has significant advantages to dither colour image with a limited colour palette. The experiment results also shows that the proposed TMT as presented in Table V is faster to dither colour image than DWT.

V. CONCLUSION

A mobile device spends significant computing power to display true colour images. An image dithering technique is an alternative approach that offers higher visual quality on image display using limited colours. Image dithering based on Tchebichef moment provides efficient computation with simple mathematical framework. The experimental results show that Tchebichef moment produces better dithering output than Floyd Steinberg. At the same time, TMT dithers faster than DWT. Hence, 2×2 TMT is ideal for high fidelity colour image dithering.

REFERENCES

[1] F. Ernawan, N.A. Abu and H. Rahmalan, "Tchebichef Moment Transform on Image Dithering for Mobile Applications" International Conference on Digital Image Processing (ICDIP 2012), Proceeding of the SPIE, Apr. 2012.

[2] R. Mukundan, S.H. Ong and P.A. Lee, "Image Analysis by Tchebichef Moments," IEEE transaction on Image Processing, Vol. 10, No. 9, Sep. 2001, pp. 1357-1364.

[3] N.A. Abu, W.S. Lang, and S. Sahib, "Image Super-Resolution via Discrete Tchebichef Moment," Proceedings of International Conference on Computer Technology and Development (ICCTD 2009), Vol. 2, Nov. 2009, pp. 315–319.

[4] N.A. Abu, N. Suryana and R. Mukundan, "Perfect Image Reconstruction Using Discrete Orthogonal Moments," Proceeding of The 4th International Conference on Visualization, Imaging and Image Processing (VIIP2004), Sep. 2004, Marbella, SPAIN, pp. 903-907.

[5] N.A. Abu, W.S. Lang, and S. Sahib, "Image Projection Over The Edge," International Conference on Industrial and Intelligent Information (ICIII 2010), Proceedings 2nd International Conference on Computer and Network Technology (ICCNT2010), Apr. 2010, pp. 344-348.

[6] F. Ernawan, E. Noersasongko and N.A. Abu "An Efficient 2×2 Tchebichef Moments for Mobile Image Compression," International Symposium on Intelligent Signal Processing and Communication System (ISPACS 2011), Dec. 2011, pp. 001-005.

[7] N.A. Abu, W.S. Lang, N. Suryana, and R. Mukundan, "An Efficient Compact Tchebichef moment for Image Compression," 10th International Conference on Information Science, Signal Processing and their applications (ISSPA2010), May 2010, pp. 448-451.

[8] W.S. Lang, N.A. Abu, and H. Rahmalan, "Fast 4x4 Tchebichef Moment Image Compression. Proceedings International Conference of Soft Computing and Pattern Recognition (SoCPaR2009), Dec. 2009, pp. 295-300.

[9] H. Rahmalan, N. A. Abu and W.S. Lang, "Using Tchebichef Moment for Fast and Efficient Image Compression," Pattern Recognition and Image Analysis, Vol. 20, No. 4, Mar. 2010, pp. 505-512.

[10] F. Ernawan, N. A. Abu and N. Suryana, "Spectrum Analysis of Speech Recognition via Discrete Tchebichef Transform," Proceedings of International Conference on Graphic and Image Processing (ICGIP 2011), Proceeding of the SPIE, Vol. 8285, No. 1, Oct. 2011, pp. 82856L-82856L-8.

[11] F. Ernawan and N.A. Abu, "Efficient Discrete Tchebichef on Spectrum Analysis of Speech Recognition," International Journal of Machine Learning and Computing, Vol. 1, No. 1, Apr. 2011, pp. 001-006.

[12] F. Ernawan, E. Noersasongko and N.A. Abu "Using Discrete Tchebichef Transform on Speech Recognition," 4th International Conference on Machine Vision (ICMV 2011), Proceeding of the SPIE, Vol. 8350, Dec. 2011, pp. 83501J-83501J-5.

[13] F. Ernawan, E. Noersasongko and N.A. Abu "Fast Dynamic Speech Recognition via Discrete Tchebichef Transform," International Conference on Informatics and Computational Intelligence (ICI 2011), Dec. 2011, pp. 250-255.

[14] Z. Ye, H. Mohamadian and Y. Ye, "Information Measures for Biometric Identification via 2D Discrete Wavelet Transform," Proceeding of the 3rd International Conference on Automation Science and Engineering, Sep. 2007, pp. 835-840.

[15] O. Cosma, "Image Dithering Based on The Wavelet Transform," Proceeding of the International Conference on Theory and Applications of Mathematics and Informatics (ICTAMI 2004), pp. 096-104.

[16] D. Malyszko and S.T. Wierzchon, "Standard and Genetic K-means Clustering Techniques in Image Segmentation," International Conference on Computer Information Systems and Industrial Management Applications (CISIM 2007), June 2007, pp. 299-304.

[17] R.W. Floyd and L. Steinberg, "An Adaptive algorithm for Spatial Greyscale," International Symposium Digest of Technical Papers, Society for Information Displays,1975, pp. 036-037.