2011 International Symposium on Intelligent Signal Processing and Communication Systems (ISPACS) December 7-9, 2011

An Efficient 2x2 Tchebichef Moments for Mobile Image Compression

Ferda Ernawan, Edi Noersasongko
Faculty of Information and Communication Technology
Universitas Dian Nuswantoro (UDINUS)
Semarang, Indonesia
e-mail: ferda1902@gmail.com, rektor@dinus.ac.id

Nur Azman Abu
Faculty of Information and Communication Technology
Universiti Teknikal Malaysia Melaka (UTeM)
Melaka, Malaysia
e-mail: nura@utem.edu.my

***Abstract* – Currently mobile digital image applications transmit a lot of images back and forth. Image compression is needed to reduce transmission payload at the expense of lower quality. At the same time, mobile devices are only expected to be equipped with lower computing power and storage. They need an efficient compression scheme especially for small images. The standard JPEG using discrete Cosine transform is a popular lossy image compression. Alternatively, this paper introduces 2x2 Tchebichef moments transform for the efficient image compression. In the previous research, larger sub-block discrete Tchebichef moments have been used extensively for image compression. The comparisons between JPEG compression and 2x2 Tchebichef moments image compressions shall be done. The preliminary experiment results show that 2x2 Tchebichef moments transform has the potential to easily perform better than JPEG image compression. The 2x2 Tchebichef moments provides an efficient and compact support for image compression.**

Keyword – Image compression; JPEG compression; discrete Cosine transforms; orthogonal moments functions; Tchebichef moments transform.

I. INTRODUCTION

Currently, popular small computing devices such as mobile phones require a lot of image transmission and processing. Image compression in mobile application has being rapidly developed to reduce the transmission for its consuming bit per pixel. This paper describes a novel image compression technique for lower image reconstruction error under lower computational requirement on small computing devices.

JPEG image compression using Discrete Cosine Transform (DCT) is a lossy image compression technique [1]. DCT has energy compaction property as it requires only real numbers. The disadvantage of using DCT image compression is the high loss of quality in compressed images. This problem is more notable at higher compression ratios [2]. Another major problem associated with the block-based DCT compression is that the decoded images manifest visually objectionable artifacts. One of the well-known artifacts in low bit rate transform-coded images is the blocking effect, which is noticeable in the form of undesired visible block boundaries [3].

This paper proposes the use of a 2x2 forward discrete Tchebichef moments transform instead of image compression. The Tchebichef Moments Transform (TMT) is a method based on discrete Tchebichef polynomials [4]. Originally, the discrete Tchebichef moment does not involve any numerical approximations and the discrete basis set is orthogonal in the integer domain of the image coordinate. Unlike continuous moments, the discrete Tchebichef orthogonal moments have a unit weight and algebraic recursive relation that is ideally suited for square image of size NxN pixels.

In the previous research, discrete Tchebichef moments generate smaller reconstruction errors than DCT when the input image was overlapped by 2 pixels [5]. TMT enlarge better quality of super sampling image when the input pixel image sub block is shifted by 2 pixels. This super-sampling technique has shown to produce smaller reconstruction difference and better images by inspecting the image quality visually. Therefore, in this experiment, the 2x2 TMT has been proposed here to get better efficiency in terms of average bit length of Huffman code on image reconstruction per 2x2 sub-block. In the previous work, orthogonal Tchebichef polynomial set has also been applied in speech recognition, several computer vision and image processing application. For examples, they are used in speech recognition [6]-[8], image analysis [4], image reconstruction [9] and image compression [10][11].

The organization of this paper is as follows. The definition of the discrete cosine transform and discrete Tchebichef polynomials are given in the next section. Section III presents the implementation of 8x8 DCT and 2x2 Tchebichef moments transform for image compression. The preliminary experiment results are discussed in Section IV and conclusion is in Section V.

II. DISCRETE COSINE TRANSFORM AND TCHEBICHEF POLYNOMIALS

A. Discrete Cosine Transform

The 2-D Discrete Cosine Transform is a popular transform used in JPEG compression standard. The image input is divided into 8x8 blocks of image data. DCT is used to

978-1-4577-2166-3/11/$26.00 ©2011 IEEE

transform each pixel in the 8x8 block into frequency domain. The definition of forward 2-D DCT on an 8x8 block of pixel $f(x,y)$ for $(x,y = 0,1,\dots,7)$ is given as follow:

$$A(u,v) = C(u)C(v)\sum_{m=0}^{7}\sum_{n=0}^{7} f(x,y)cos\left(\frac{\pi(2m+1)u}{16}\right)cos\left(\frac{\pi(2n+1)v}{16}\right) \quad (1)$$

for $u = 0,1,\dots,7$ and $v = 0,1,\dots,7$

where $f(x,y)$ is the input image, $A(u,v)$ is the DCT coefficients and

$$C(u) = \begin{cases} \frac{1}{\sqrt{M}}, & u = 0 \\ \sqrt{\frac{2}{M}}, & u > 0 \end{cases} \qquad C(v) = \begin{cases} \frac{1}{\sqrt{N}}, & v = 0 \\ \sqrt{\frac{2}{N}}, & v > 0 \end{cases} \quad (2)$$

The two dimensional DCT can be computed using the one dimensional DCT horizontally and then vertically across the signal. The inverse of Discrete Cosine Transform is given as follows:

$$f(x,y) = \sum_{m=0}^{7}\sum_{n=0}^{7} C(u)C(v)\,A(u,v)cos\left(\frac{\pi(2m+1)u}{16}\right)cos\left(\frac{\pi(2n+1)v}{16}\right) \quad (3)$$

The output of transformed 8x8 blocks of image data are 64 DCT coefficients. The first coefficient $A(0,0)$ is called the DC coefficient, and another 63 coefficients are called AC coefficient.

B. *Orthogonal Tchebichef Polynomials*

For a given set $\{t_n(x)\}$ of input value (image intensity values) of size $N = 2$, the forward discrete orthogonal Tchebichef Moments of order $m + n$ is given as follows [4]:

$$T_{mn} = \frac{1}{\rho(m,N)\rho(n,N)}\sum_{x=0}^{1}\sum_{y=0}^{1} t_m(x)t_n(y)f(x,y) \quad (4)$$

for $m = 0,1$ and $n = 0,1$

Where $f(x,y)$ denote the intensity value at the pixel position (x,y) in the image. The $t_n(x)$ are defined using the following recursive relation:

$$t_0(x) = 1, \quad (5)$$

$$t_1(x) = \frac{2x+1-N}{N}, \quad (6)$$

The set $\{t_n(x)\}$ has a squared-norm given by

$$\rho(n,N) = \sum_{i=0}^{1}\{t_i(x)\}^2 \quad (7)$$

$$= \frac{N.\left(1-\frac{1^2}{N^2}\right).\left(1-\frac{2^2}{N^2}\right)}{2n+1} \quad (8)$$

where $n = 0,1$. The description of squared-norm $\rho()$ and the properties of orthogonal Tchebichef polynomials is given in [4]. The process of image reconstruction from its moments, the inverse moment Tchebichef moments are given as follows:

$$\tilde{f}(x,y) = \sum_{m=0}^{M}\sum_{n=0}^{M} T_{mn}t_m(x)t_n(y) \quad (9)$$

Where M denotes the maximum order of moments used and $\tilde{f}(x,y)$ the reconstructed intensity distribution.

III. JPEG Coding versus 2x2 TMT Compression

JPEG standard is a simple lossy compression technique. Firstly, the RGB image is converted to YCbCr. RGB is separately into a luminance part (Y) and Chrominance part (Cr and Cb). Based on JPEG compression, an image is divided into the 8x8 size blocks, and each block of the image data is transformed by 2-dimentional DCT baseline sequential coding. Next, the DCT coefficient of each block is divided with quantization table independently. As recommendation of JPEG, the Huffman code is used in here.

In our research, an image is divided into 2x2 sized blocks of image data. Each block is performed by discrete Tchebichef moments baseline sequential. The quantization table 2x2 is proposed to reduce the high frequencies. Next, Huffman code is used to calculate the average bit length of TMT coefficient. 2x2 TMT is implemented to achieve excellent compression performance than JPEG compression. The visual 2x2 TMT image compression is shown in Fig. 1.

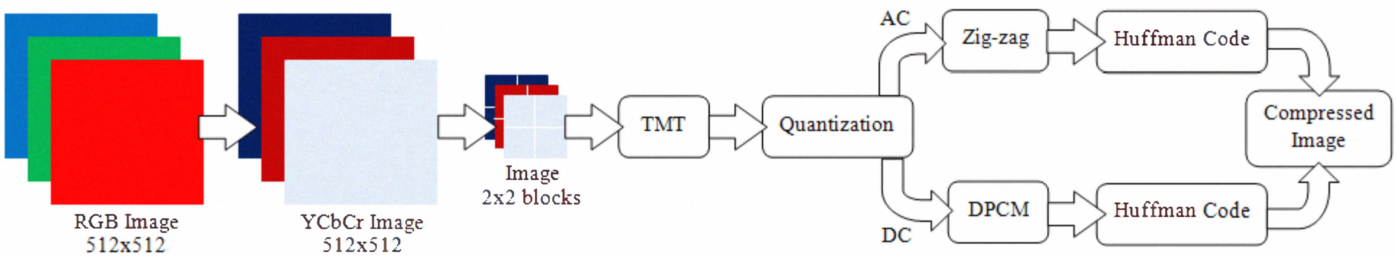

Figure 1. Visualization of Image Compression

A. RGB to YCbCr

In order to achieve a good compression performance, the correlation between the color components is reduced by converting the RGB image to YCbCr. The RGB image should be separated into a luminance (Y) and two chrominance (Cb and Cr). The advantage of converting image into luminance chrominance color space is that the luminance and chrominance components are very much dĕcor-related between each other [12]. YCbCr can be computed directly from 8 bit RGB as follows:

$$\begin{bmatrix} Y \\ Cb \\ Cr \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.1687 & -0.3313 & 0.5 \\ 0.5 & -0.4187 & -0.0813 \end{bmatrix} . \begin{bmatrix} R \\ G \\ B \end{bmatrix} + \begin{bmatrix} 0 \\ 128 \\ 128 \end{bmatrix} \quad (10)$$

The inverse transformation from YCbCr to RGB is given as follows:

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1.4021 \\ 1 & -0.34414 & -0.71414 \\ 1 & 1.7718 & 0 \end{bmatrix} . \begin{bmatrix} Y \\ Cb \\ Cr \end{bmatrix} - \begin{bmatrix} 0 \\ 128 \\ 128 \end{bmatrix} \quad (11)$$

Moreover, the chrominance channels contain much redundant color information and can easily be sub-sampled without any visual quality for the reconstructed image.

B. TMT

The image matrix was subdivided into 2x2 pixels where the orthogonal Tchebichef moments on each block are computed independently. The block size N is taken to be 2, and it is compared to the JPEG compression with $N = 8$ using DCT. Based on discrete orthogonal moments as defines in (4)-(8) above, a kernel matrix $K_{(2x2)}$ is given as follows:

$$K = \begin{bmatrix} t_0(0) & t_1(0) \\ t_0(1) & t_1(1) \end{bmatrix} \quad (12)$$

The image block matrix by $F_{(2x2)}$ with $f(x, y)$ denotes the intensity value of the pixel:

$$F = \begin{bmatrix} f(0,0) & f(0,1) \\ f(1,0) & f(1,1) \end{bmatrix} \quad (13)$$

The matrix $T_{(2x2)}$ of moments is defined based on (4) above as follows:

$$T = K^T F K \quad (14)$$

This process is repeated for every block in the original image to generate coefficient discrete orthogonal Tchebichef Moments. The inverse moments relation used to reconstruct the image block from the above moments is given as follow:

$$G = K T K^T \quad (15)$$

Where $G_{(2x2)}$ denotes the matrix image of the reconstructed intensity value. This process is repeated for every block of the coefficient Tchebichef Moments.

C. Quantization

The purpose of the quantization is to remove the high frequencies or discards information which is not visually significant. This is done by dividing the coefficient transform in each block using quantization matrix $Q(u, v)$. This process removes the high frequencies present in the original image. The quantization matrix is defined into two quantization matrices, one for luminance and another for chrominance. The quantization table luminance Q_{CL} and chrominance Q_{CR} below for DCT image compression is defined mathematically as follows:

$$Q_{CL} = \begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix} \quad (16)$$

$$Q_{CR} = \begin{bmatrix} 17 & 18 & 24 & 47 & 99 & 99 & 99 & 99 \\ 18 & 21 & 26 & 66 & 99 & 99 & 99 & 99 \\ 24 & 26 & 56 & 99 & 99 & 99 & 99 & 99 \\ 47 & 66 & 99 & 99 & 99 & 99 & 99 & 99 \\ 99 & 99 & 99 & 99 & 99 & 99 & 99 & 99 \\ 99 & 99 & 99 & 99 & 99 & 99 & 99 & 99 \\ 99 & 99 & 99 & 99 & 99 & 99 & 99 & 99 \\ 99 & 99 & 99 & 99 & 99 & 99 & 99 & 99 \end{bmatrix} \quad (17)$$

In the propose quantization tables of this research, the moments coefficients are divided separately with the quantization table for luminance Q_{ML} and chrominance Q_{MR} below:

$$Q_{ML} = \begin{bmatrix} 8 & 16 \\ 16 & 24 \end{bmatrix} \quad (18)$$

$$Q_{MR} = \begin{bmatrix} 8 & 16 \\ 16 & 32 \end{bmatrix} \quad (19)$$

The Quantization steps are defined as follow by rounding to the nearest integer:

$$T_q(u, v) = Round(\frac{T(u, v)}{Q(u, v)}) \quad (20)$$

Dequantization is the inverse function by multiplying the coefficient in each block using the same quantization matrix $Q(u, v)$. The aim is to compress the image as much as possible without visible artifacts. The dequantization is defined as follows:

$$T_{q'}(i, j) = T_q(u, v) Q(u, v) \quad (21)$$

This process is used to remove the high frequencies in the original image. The quantization process contains a large number of zeroes as a result of the filtering of high frequency noise.

D. Zig-zag order

After the quantization process, the DC coefficient is separated with the AC coefficient. The quantized DC coefficient is encoded as the difference from the DC term of the previous block in the encoding order. For AC coefficient, each sub-block is arranged in a zig-zag to order the AC coefficient into a linear array. The implementation of zig-zag order for JPEG compression and 2x2 Tchebichef moments image compressions is shown on the left and right of Fig. 2 respectively.

0	1	5	6	14	15	27	28
2	4	7	13	16	26	29	42
3	8	12	17	25	30	41	43
9	11	18	24	31	40	44	53
10	19	23	32	39	45	52	54
20	22	33	38	46	51	55	60
21	34	37	47	50	56	59	61
35	36	48	49	57	58	62	63

0	1
2	3

Figure 2. Zig-zag order of 8x8 DCT (left) and 2x2 TMT (right).

The zig-zag output of quantized AC coefficient is represented as a sequence of a data set. Next, run length encoding is used to reduce the size of a repeating coefficient value in the sequence a set coefficient data. Run length coding is a simple technique to coefficient value when there is a long run of the same value. The coefficient value can be represented compactly by simply indicating the coefficient value and the length of its run when it appears. The output of run length coding represents a value of the pixel as symbols and the length of occurrence of the symbols. The symbols and variable length of occurrence are used in Huffman coding to retrieve code words and their length of code words.

E. Huffman Coding

Huffman coding is a coding technique to produce the shortest possible average code length of the source symbol set and the probability of occurrence of the symbols [13]. Using these probability values, a set of Huffman code of the symbols can be generated by Huffman Tree. Next, the average bit length score is calculated to find the average bit length of DC and AC coefficient. The Huffman codes are stored in the Huffman Table.

For image with three components, the encoder can store four sets of Huffman table (AC tables for Luminance and Chrominance and DC tables for Luminance ad Chrominance). Huffman tables used during the compression process are stored as header information in the compressed image file in order to uniquely decode the coefficients during the decompression process [13].

F. Image Measurement Quality

The image reconstruction error calculated for the differences between reconstructed image $g(i,j,k)$ and original image $f(i,j,k)$ is given as follows:

$$E(S) = \frac{1}{3MN}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\sum_{k=0}^{2}|g(i,j,k) - f(i,j,k)| \qquad (22)$$

where the original image size is MxN and the third index refers to the value of three colors RGB samples. Another convenient measurement is the Means Squared Error (MSE), it calculates the average of the square of the error. The MSE is defined as follows:

$$MSE = \sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\sum_{k=0}^{2}\|g(i,j,k) - f(i,j,k)\|^2 \qquad (23)$$

For evaluation of the propose method, Peak Signal to Noise Ratio (PSNR) is used as an objectives measurement for the performance. PSNR is used to measure the quality of image reconstruction. A higher PSNR means that the reconstruction is higher in quality. The PSNR is defined as follows:

$$PSNR = 20\log 10\left(\frac{Max_i}{\sqrt{MSE}}\right) \qquad (24)$$

$$= 10\log 10\left(\frac{255^2}{\sqrt{MSE}}\right) \qquad (25)$$

where Max_i is the maximum possible pixel value of the image. In this experiment, the sample image is represented by using 8 bits per sample, its means that the maximum intensity of sample image is 255.

IV. Experiment Results

The Average bit length DC coefficient of Huffman Code between 8x8 DCT and 2x2 TMT have been calculated. 2x2 image block of image compression using TMT is more compact in average bit length of Huffman code than DCT. The comparison of average bit length DC Coefficient and AC coefficient via 8x8 DCT and 2x2 TMT respectively are presented in Table 1 and Table 2.

Table 1. Average bit length score between 8x8 DCT and 2x2 TMT

Average bit length	8x8 DCT	2x2 TMT
DC Luminance	5.244873	3.436386
DC Chrominance Red (Cr)	3.908691	2.063507
DC Chrominance Blue (Cb)	3.993164	2.153931

Table 2. Average bit length score between 8x8 DCT and 2x2 TMT

Average bit length	8x8 DCT	2x2 TMT
AC Luminance	2.79739	2.705377
AC Chrominance Red (Cr)	2.152667	1.965315
AC Chrominance Blue (Cb)	2.116016	2.056809

As the result, the redundancy will be very low if the average bit length required is minimum bit. The experiment results show 2x2 TMT produce minimum score average bit length in DC coefficient or AC coefficient. The comparison quality of image reconstruction between JPEG compression and 2x2 TMT image compressions is shown in Table 3.

Table 3. Average Error Score between 8x8 DCT and 2x2 TMT.

Image Measurement	8x8 DCT	2x2 TMT	Diff.
Full Error	10.0374	5.7360	4.3014
MSE	181.7553	53.1564	128.5989
PSNR	25.5359	30.8752	5.3393

In order to observe the effectiveness of 2x2 TMT, the reconstruction image is zoomed in to 400%. The experiment results of image compression using 8x8 DCT and 2x2 TMT are shown in Fig. 4.

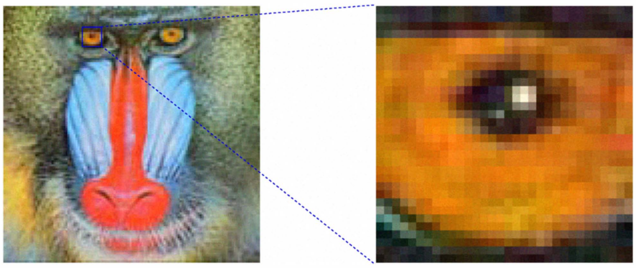

Figure 3. Original Image (left) and zoomed in image to 400% (right).

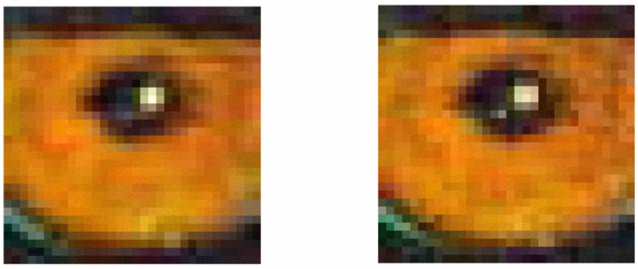

Figure 4. The comparison between 8x8 DCT (left) and 2x2 TMT (right) on zoomed in image to 400%.

V. COMPARATIVE ANALYSIS

In this experiment, the right eye of the original image is zoomed in to 400% and evaluated as presented on the right of Fig. 3. Based on the left pictures of Fig. 4, image compression using 8x8 DCT produces artifact image and it gives blur image output when the image data was zoomed in to 400%. While the observation as presented on the right of Fig. 4, 2x2 TMT image compressions produce almost the same as the original image on the right of Fig. 3.

The experiment result as presented in Fig. 4 showed that image compression using 2x2 TMT indicated a better quality than the standard JPEG compression which produce artifact image. 2x2 TMT also produces minimum score average bit length of Huffman code than 8x8 DCT as shown in Table 1 and Table 2. According to observation as presented in Table 3, 2x2 TMT image compressions provide a minimum error reconstruction and produce a higher quality of the image reconstruction than JPEG compression. In this experiment, the current issue of Tchebichef moments was a compromise between smaller sub block of image data and table quantization. The proposed method achieved high quality image compression and minimum score of average bit length of Huffman code.

VI. CONCLUSION

In the previous research, discrete Tchebichef moments have been used in image compression. By using the 2x2 sub block of image, TMT give a significant advantage in image compression performance. JPEG has been a popular image compression over the last couple of decades. The developments of image compression onwards mainly focus on larger higher quality image. In this paper a simplified matrix of 2x2 TMT is introduced for compression on small images. The 2x2 TMT produces higher image quality with lower error image reconstruction. The image output is almost the same as the original image. The experiment results show that 2x2 TMT has the potential to perform well in terms of the quality of image reconstruction and average bit length of Huffman code. Even though this 2x2 TMT may not give better image compression in general, it is very suitable for compression using mobile computing devices on small images.

REFERENCES

[1] G. K. Wallace, "The JPEG Still Picture Compression Standard," *Communication of the ACM*, Vol. 34, No. 4, Apr. 1991, pp. 30-44, 1991.

[2] A. Khashman and K. Dimililer, "Neural Network Arbitration for Optimum DCT Image Compression," *International Conference on Computer as a Tool*, Dec. 2007, pp. 151-156.

[3] K. Shimauchi, M. Ogawa and A. Taguchi, "JPEG Based Image Compression with Adaptive Resolution Conversion System," *International Symposium on Circuit and System*, Vol. 5, Sep. 2004, pp. 437-440.

[4] R. Mukundan, S. H. Ong and P. A. Lee "Image Analysis by Tchebichef Moments," *IEEE transaction on Image Processing*, Vol. 10, No. 9, Sep. 2001, pp. 1357-1364.

[5] N. A. Abu, W.S. Lang and S. Sahib, "Image Super-Resolution via Discrete Tchebichef Moment," *Proceedings of International Conference on Computer Technology and Development (ICCTD 2009)*, Vol. 2, Nov. 2009, pp. 315–319.

[6] F. Ernawan, N. A. Abu and N. Suryana, "Spectrum Analysis of Speech Recognition via Discrete Tchebichef Transform," *Proceedings of International Conference on Graphic and Image Processing* (ICGIP 2011), SPIE, Vol. 8285 No. 1, Oct. 2011.

[7] F. Ernawan, N. A. Abu and N. Suryana, "The Efficient Discrete Tchebichef Transform for Spectrum Analysis of Speech Recognition," *Proceedings 3rd International Conference on Machine Learning and Computing*, Vol. 4, Feb. 2011, pp. 50-54.

[8] F. Ernawan and N.A. Abu "Efficient Discrete Tchebichef on Spectrum Analysis of Speech Recognition," *International Journal of Machine Learning and Computing*, Vol. 1, No. 1, Apr. 2011, pp. 1-6.

[9] R. Mukundan, "Improving Image Reconstruction Accuracy Using Discrete Orthonormal Moments," *Proceeding of International Conference on Imaging Systems, Science and Technology*, June 2003, pp. 287-293.

[10] H. Rahmalan, N. A. Abu and W.S. Lang, "Using Tchebichef Moment for Fast and Efficient Image Compression," *Pattern Recognition and Image Analysis*, Vol. 20, No. 4, Mar. 2010, pp. 505-512.

[11] W.S. Lang, N.A. Abu and H. Rahmalan, "Fast 4x4 Tchebichef Moment Image Compression," *International Conference of Soft Computing and Pattern Recognition*, Dec. 2009, pp. 295-300.

[12] Tinku Acharya and Ping-Sing Tsai, "JPEG2000 Standard for Image Compression: Concepts, Algorithm and VLSI architectures," *John Wiley and Sons*, 2005, pp. 74.

[13] D. A. Huffman, "A Method for the Construction of Minimum-Redundancy Codes," *Proceedings of the IRE,* Vol. 40, No. 9, Sep. 1952, pp. 1098-1101.