Quantitative Analysis for Management

Linear Programming Models:

Linear Programming Problem

- 1. Tujuan adalah maximize or minimize variabel dependen dari beberapa kuantitas variabel independen (fungsi tujuan).

- 2. Batasan-batasan yang diperlukan guna mencapai tujuan.
- Tujuan dan Batasan dinyatakan dalam persamaan linear.

Basic Assumptions of Linear Programming

- Certainty
- Proportionality
- Additivity
- Divisibility
- Nonnegativity

Flair Furniture Company Data - Table 7.1

Hours Required to Produce One Unit

Department	X_1 Tables	X_2 Chairs	Available Hours This Week
Carpentry	4	3	240
Painting/Varnishing	2	1	100
Profit/unit	$7	$5	

Flair Furniture Company Data - Table 7.1

STEP 1:

Objective: **Maximize:** $7X_1 + 5X_2$

STEP 2:

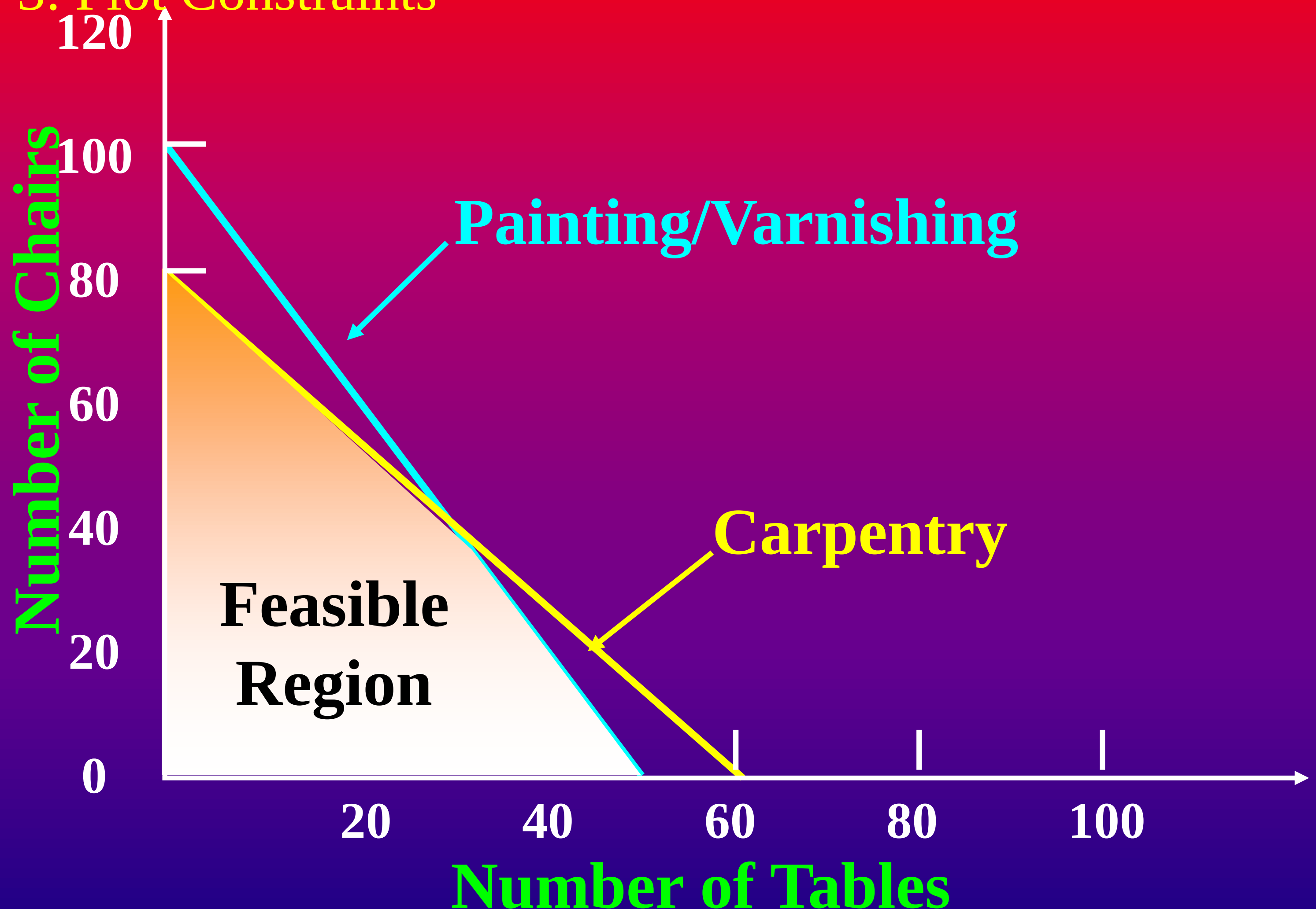
Constraints:

$4X_1 + 3X_2 \le 240$ **(carpentry)**

$2X_1 + 1X_2 \le 100$ **(painting & varnishing)**

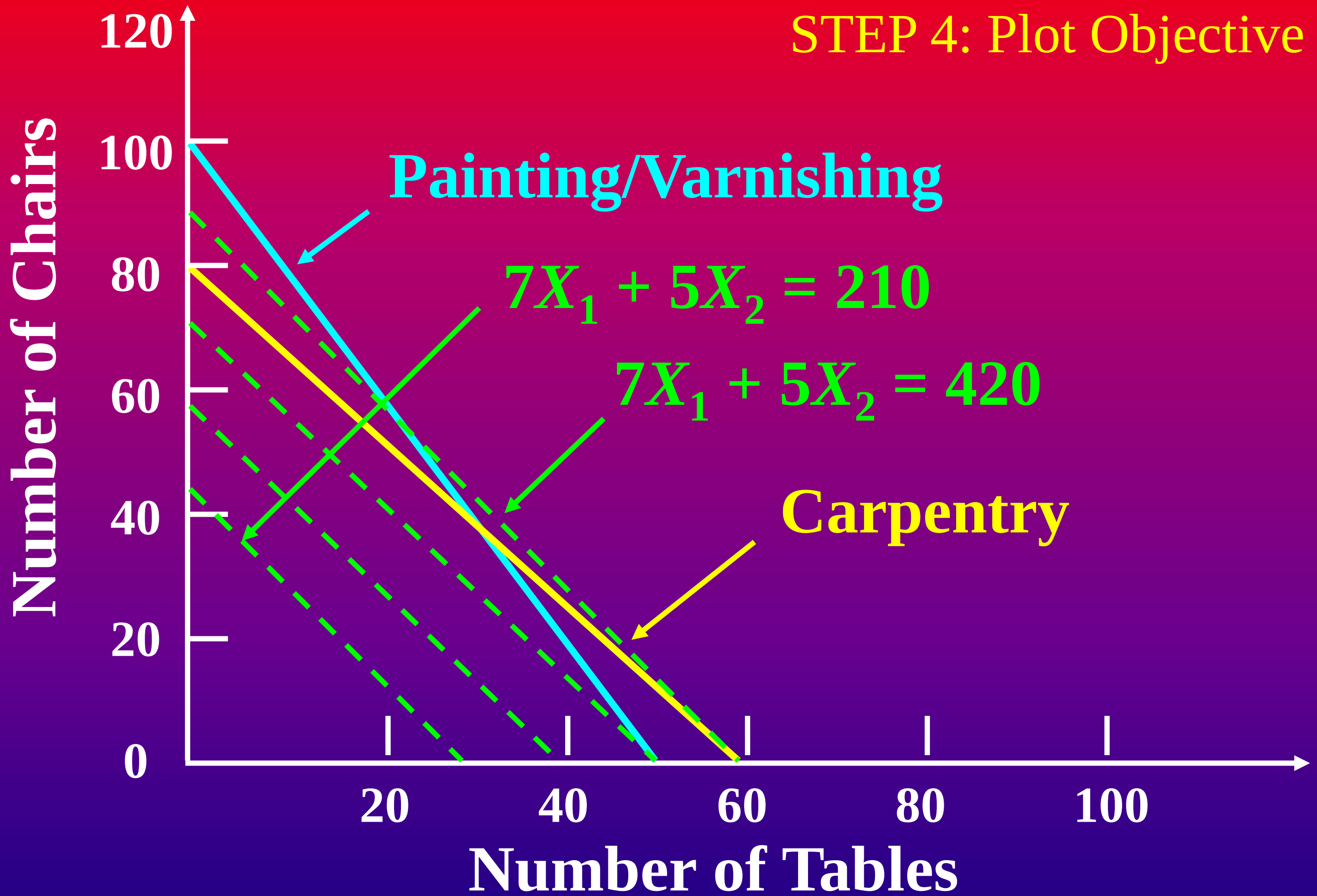
Flair Furniture Company Feasible Region

STEP 3: Plot Constraints



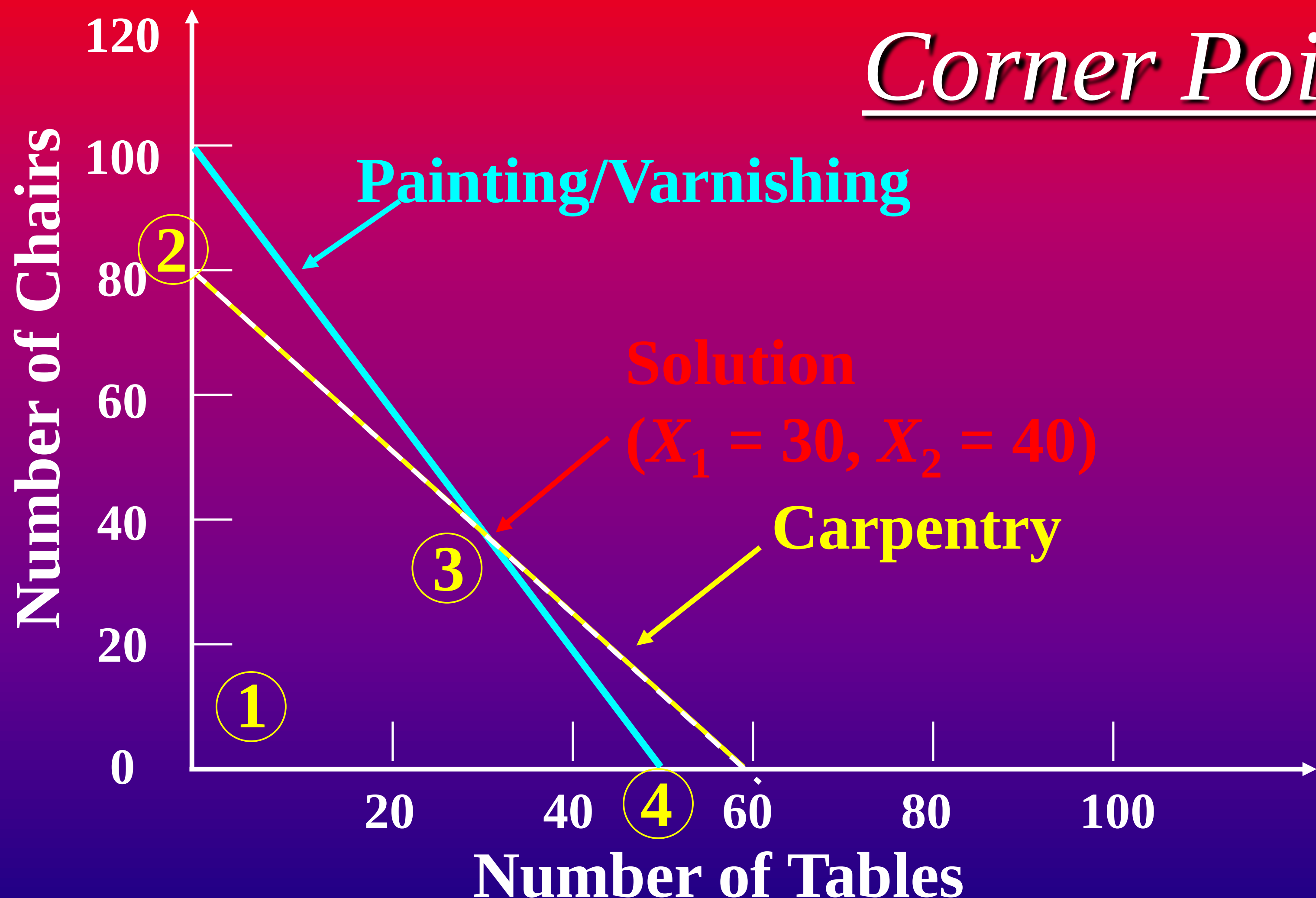
Flair Furniture Company Isoprofit Lines

STEP 4: Plot Objective Function



Flair Furniture Company Optimal Solution

Corner Points



Test Corner Point Solutions

Point 1) (0,0) => 7(0) + 5(0) = $0

Point 2) (0,100) => 7(0) + 5(80) = $400

Point 3) (30,40) => 7(30) + 5(40) = $410

Point 4) (50,0) => 7(50) + 5(0) = $350

Solve Equations Simultaneously

To get X1 & X2 values for Point 3:

$4X1 + 3X2 <= 240$

$X1 = 60 - 3/4\ X2$

$2X1 + 1X2 <= 100$

$X1 = 50 - 1/2\ X2$

$60 - 3/4\ X2 = 50 - 1/2\ X2$

$60 - 50 = 3/4\ X2 - 1/2\ X2$

$10 = 1/4\ X2$

$\boxed{40 = X2;}$

so, $4X1 + 3(40) = 240$

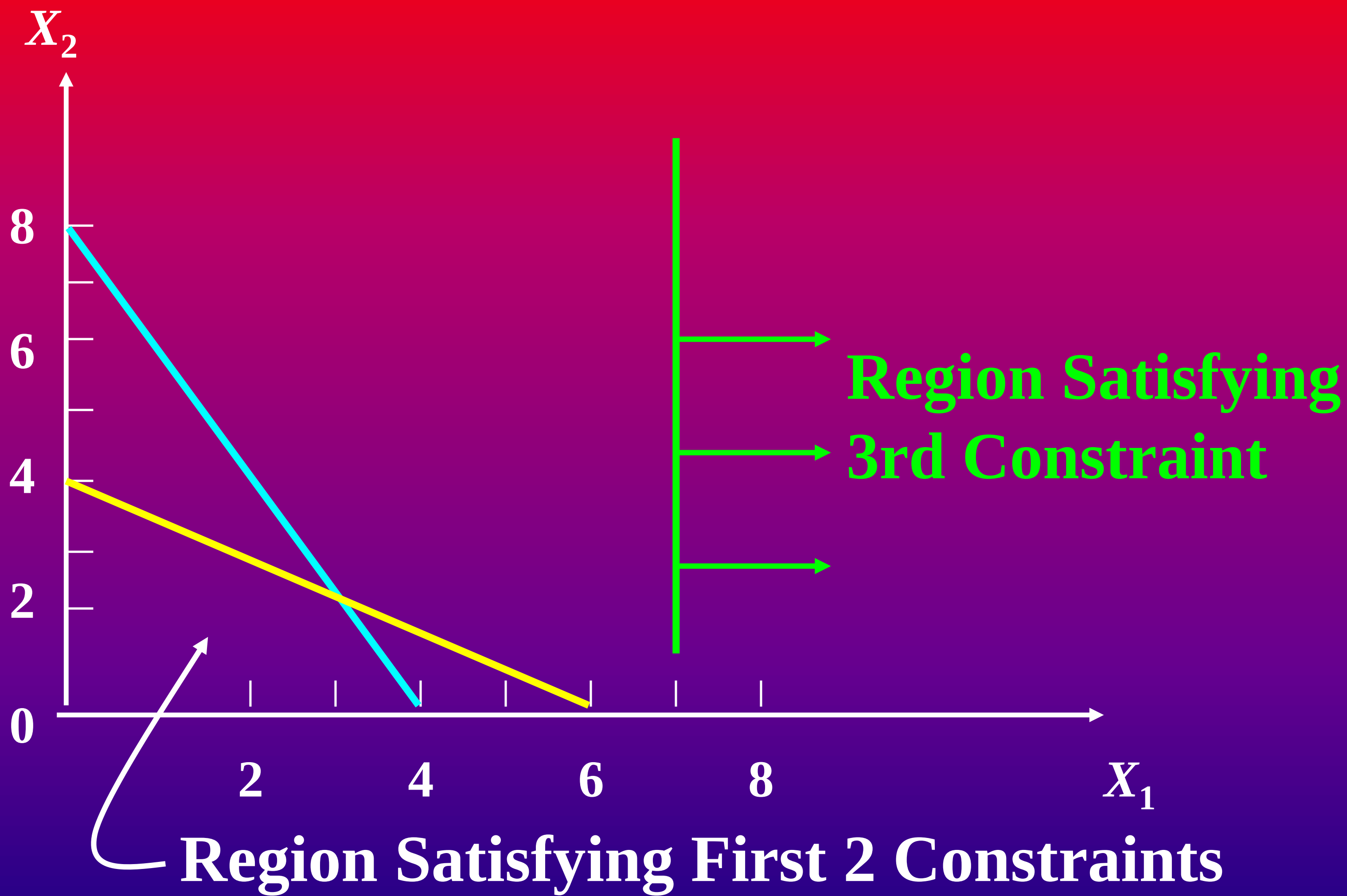
$4X1 = 240 - 120$

$\boxed{X1 = 30}$

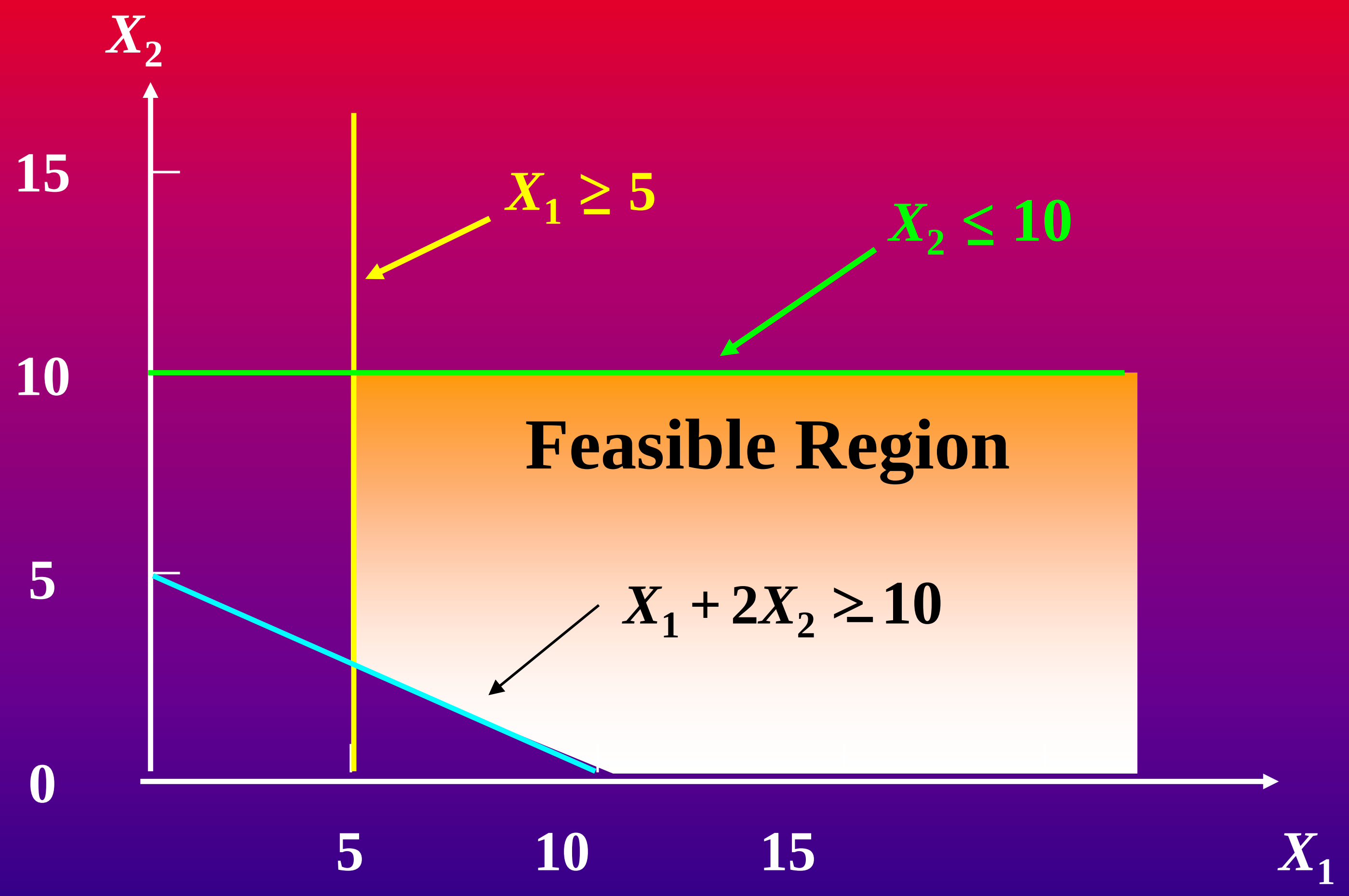
Special Cases in LP

- Infeasibility
- Unbounded Solutions
- Redundancy
- Degeneracy
- More Than One Optimal Solution

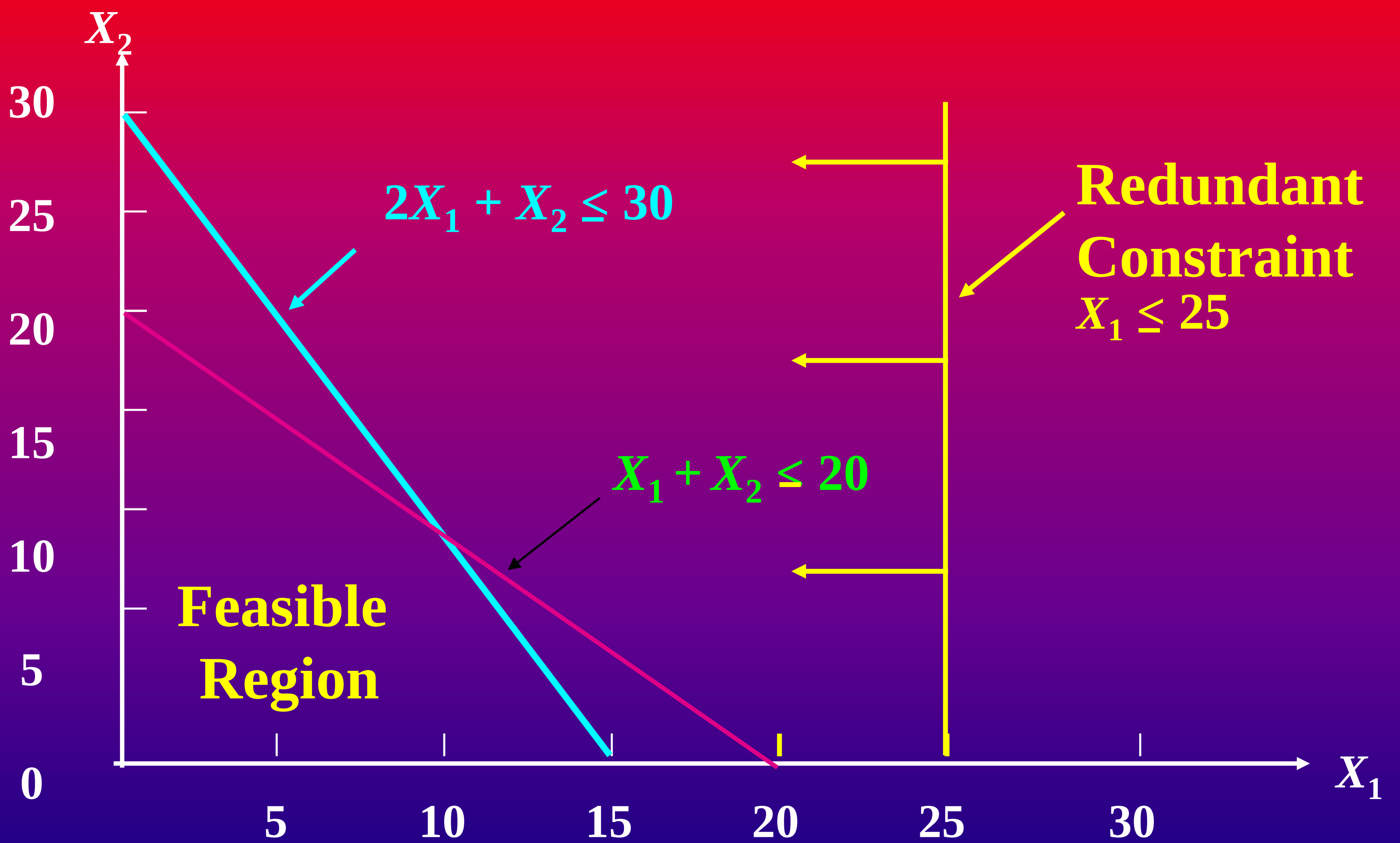
A Problem with No Feasible Solution



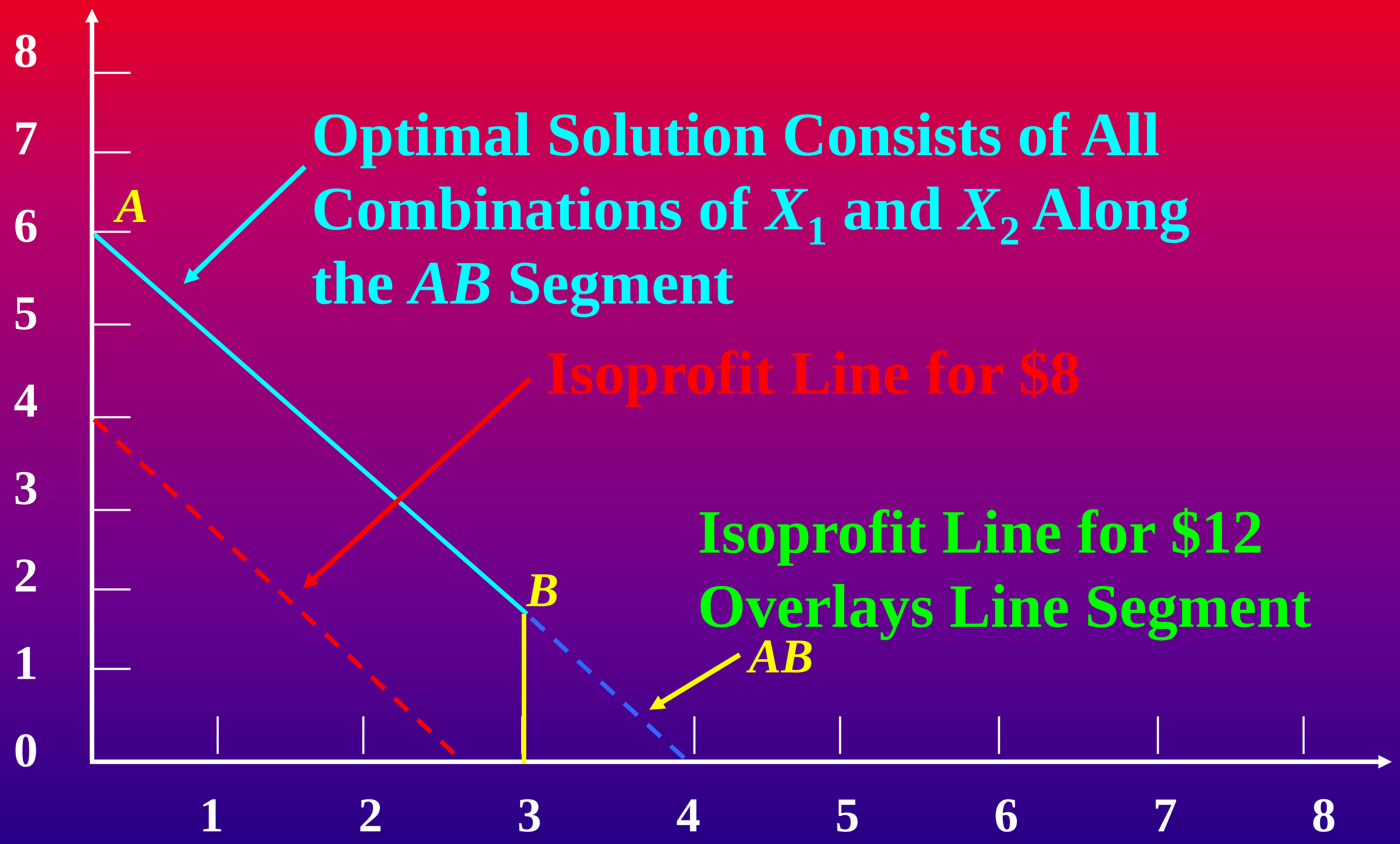
A Solution Region That is Unbounded to the Right



A Problem with a Redundant Constraint



An Example of Alternate Optimal Solutions



Marketing Applications

Media Selection - Win Big Gambling Club

Medium	Audience Reached Per Ad	Cost Per Ad($)	Maximum Ads Per Week
TV spot (1 minute)	5,000	800	12
Daily newspaper (full-page ad)	8,500	925	5
Radio spot (30 seconds, prime time)	2,400	290	25
Radio spot (1 minute, afternoon)	2,800	380	20

Win Big Gambling Club

Maximize: $5000X_1 + 8500X_2 + 2400X_3 + 2800X_4$

Subject to:

$X_1 \leq 12$ **(max TV spots/week)**

$X_2 \leq 5$ **(max newspaper ads/week)**

$X_3 \leq 25$ **(max 30-sec. radio spots/week)**

$X_4 \leq 20$ **(max 1-min. radio spots/week)**

$800X_1 + 925X_2 + 290X_3 + 380X_4 \leq 8000$ **(weekly ad budget)**

$X_3 + X_4 \geq 5$ **(min radio spots/week)**

$290X_3 + 380X_4 \leq 1800$ **(max radio expense)**

Manufacturing Applications

Production Mix - Fifth Avenue

Variety of Tie	Selling Price per Tie ($)	Monthly Contract Minimum	Monthly Demand	Material Required per Tie (Yds)	Material Require ments
All silk	6.70	6000	7000	0.125	100% silk
All polyester	3.55	10000	14000	0.08	100% polyester
Poly-cotton blend 1	4.31	13000	16000	0.10	50% poly/50% cotton
Poly-cotton - blend 2	4.81	6000	8500	0.10	30% poly/70% cotton

Fifth Avenue

Maximize: $4.08X_1 + 3.07X_2 + 3.56X_3 + 4.00X_4$

Subject to:

$0.125X_1 \leq 800$ **(yards of silk)**

$0.08X_2 + 0.05X_3 + 0.03X_4 \leq 3000$ **(yards polyester)**

$0.05X_3 + 0.07X_4 \leq 1600$ **(yards cotton)**

$X_1 \geq 6000$ **(contract min, silk)** $X_1 \leq 7000$ **(contract max, silk)**

$X_2 \geq 1000$ **(contract min, all polyester)**

$X_2 \leq 14000$ **(contract max, all polyester)**

$X_3 \geq 13000$ **(contract min, blend1)** $X_3 \leq 16000$ **(contract max, blend1)**

$X_4 \geq 6000$ **(contract min, blend2)** $X_4 \leq 8500$ **(contract max, blend 2)**

Manufacturing Applications

Truck Loading - Goodman Shipping

Item	Value ($)	Weight (lbs)
1	22,500	7,500
2	24,000	7,500
3	8,000	3,000
4	9,500	3,500
5	11,500	4,000
6	9,750	3,500

Goodman Shipping

Maximize load value : $22500\ X_1 + 24000\ X_2 + 8000\ X_3 + 9500\ X_4 + 11500\ X_5 + 9750\ X_6$

Subject to :

$7500\ X_1 + 7500\ X_2 + 3000\ X_3 + 3500\ X_4 + 4000\ X_5 + 3500\ X_6 \leq 10000$ **(Capacity)**

$X_1 \leq 1$

$X_2 \leq 1$

$X_3 \leq 1$

$X_4 \leq 1$

$X_5 \leq 1$

$X_6 \leq 1$

Flair Furniture Company

Hours Required to Produce One Unit

Department	X_1 Tables	X_2 Chairs	Available Hours This Week
Carpentry	4	3	240
Painting/Varnishing	2	1	100
Profit/unit	\$7	\$5	

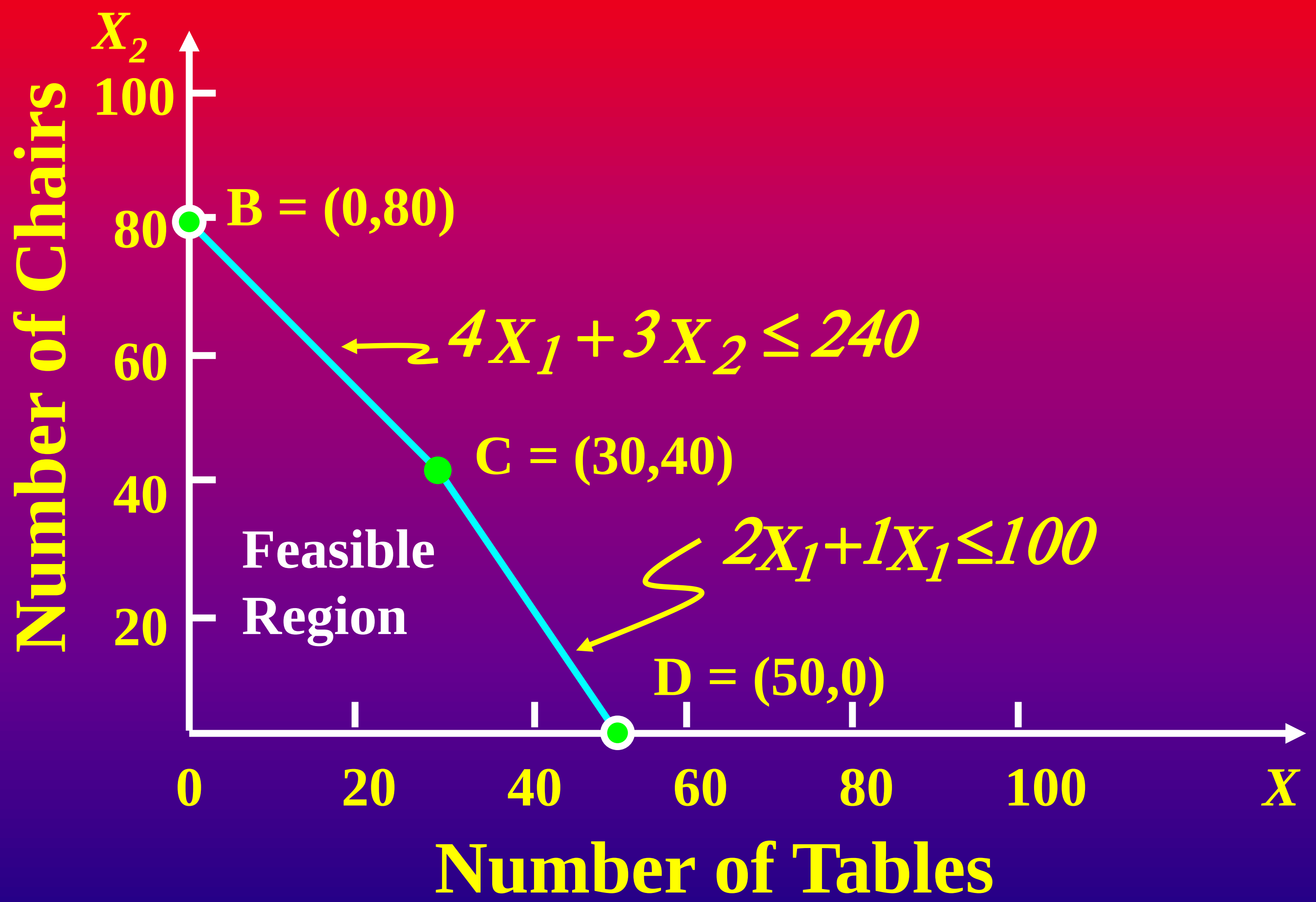
Constraints:

$$4X_1 + 3X_2 \le 240 \quad \text{(carpentry)}$$

$$2X_1 + 1X_2 \le 100 \quad \text{(painting \& varnishing)}$$

Objective: **Maximize:** $7X_1 + 5X_2$

Flair Furniture Company's Feasible Region & Corner Points



Flair Furniture - Adding Slack Variables

Constraints:

$$4X_1 + 3X_2 \le 240 \text{ (carpentry)}$$

$$2X_1 + 1X_2 \le 100 \text{ (painting \& varnishing)}$$

Constraints with Slack Variables

$$4X_1 + 3X_2 + S_1 = 240 \text{ (carpentry)}$$

$$2X_1 + 1X_2 + S_2 = 100 \text{ (painting \&varnishing)}$$

Objective Function

$$7X_1 + 5X_2$$

Objective Function with Slack Variables

$$7X_1 + 5X_2 + 1S_1 + 1S_2$$

Flair Furniture's Initial Simplex Tableau

Profit per Unit Column	*Production Mix Column*	*Real Variables Columns*		*Slack Variables Columns*		*Constant Column*
C_j →		$7	$5	$0	$0	← *Profit per unit row*
↓	*Solution Mix*	X_1	X_2	S_1	S_2	*Quantity*
$0	S_1	2	1	1	0	100
$0	S_2	4	3	0	1	240
	Z_j	$0	$0	$0	$0	$0
	$C_j - Z_j$	$7	$5	$0	$0	$0

Constraint equation rows (rows S_1, S_2)

Gross profit row (Z_j)

Net profit row ($C_j - Z_j$)

Pivot Row, Pivot Number Identified in the Initial Simplex Tableau

C_j	Solution Mix	X_1	X_2	S_1	S_2	Quantity
		$7	$5	$0	$0	
$0	S_1	2	1	1	0	100
$0	S_2	4	3	0	1	240
	Z_j	$0	$0	$0	$0	$0
	$C_j - Z_j$	$7	$5	$0	$0	$0

Pivot row: S_1 row (100)

Pivot number: 2

Pivot column: X_1 column ($7)

Completed Second Simplex Tableau for Flair Furniture

C_j →		\$7	\$5	\$0	\$0	
↓	***Solution Mix***	X_1	X_2	S_1	S_2	***Quantity***
\$7	X_1	1	1/2	1/2	0	50
\$0	S_2	0	1	-2	1	40
	Z_j	\$7	\$7/2	\$7/2	\$0	\$350
	$C_j - Z_j$	\$0	\$3/2	-\$7/2	\$0	

Pivot Row, Column, and Number Identified in Second Simplex Tableau

C_j →		$7	$5	$0	$0	
↓	Solution Mix	X_1	X_2	S_1	S_2	Quantity
$7	X_1	1	1/2	1/2	0	50
$0	S_2	0	1 (Pivot number)	-2	1	40 ← Pivot row
	Z_j	$7	$7/2	$7/2	$0	$350 (Total Profit)
	$C_j - Z_j$	$0	$3/2 (Pivot column)	-$7/2	$0	

Calculating the New X_1 Row for Flair's Third Tableau

$$\begin{pmatrix}\text{Number}\\ \text{in new}\\ X_1 \text{ row}\end{pmatrix} = \begin{pmatrix}\text{Number}\\ \text{in old}\\ X_1 \text{ row}\end{pmatrix} - \left[\begin{pmatrix}\text{Number}\\ \text{above pivot}\\ \text{number}\end{pmatrix} \times \begin{pmatrix}\text{Corresponding}\\ \text{number in}\\ \text{new } X_2 \text{ row}\end{pmatrix}\right]$$

1	=	1	-	(1/2)	x	(0)
0	=	1/2	-	(1/2)	x	(1)
3/2	=	1/2	-	(1/2)	x	(-2)
-1/2	=	0	-	(1/2)	x	(1)
30	=	50	-	(1/2)	x	(40)

Final Simplex Tableau for the Flair Furniture Problem

C_j →		\$7	\$5	\$0	\$0	
↓	Solution Mix	X_1	X_2	S_1	S_2	Quantity
\$7	X_1	1	0	3/2	-1/2	30
\$5	X_2	0	1	-2	1	40
	Z_j	\$7	5	\$1/2	\$3/2	\$410
	$C_j - Z_j$	\$0	\$0	-\$1/2	-\$3/2	

Simplex Steps for Maximization

1. Choose the variable with the greatest positive $C_j - Z_j$ to enter the solution.
2. Determine the row to be replaced by selecting that one with the smallest (non-negative) quantity-to-pivot-column ratio.
3. Calculate the new values for the pivot row.
4. Calculate the new values for the other row(s).
5. Calculate the C_j and $C_j - Z_j$ values for this tableau. If there are any $C_j - Z_j$ values greater than zero, return to Step 1.

Surplus & Artificial Variables

Constraints

$$5X_1 + 10X_2 + 8X_3 \geq 210$$

$$25X_1 + 30X_2 = 900$$

Constraints-Surplus & Artificial Variables

$$5X_1 + 10X_2 + 8X_3 - S_1 + A_1 = 210$$

$$25X_1 + 30X_2 + A_2 = 900$$

Objective Function

Min: $5X_1 + 9X_2 + 7X_3$

Objective Function-Surplus & Artificial Variables

Min : $5X_1 + 9X_2 + 7X_3 + 0S_1 + MA_1 + MA_2$

Simplex Steps for Minimization

1. Choose the variable with the greatest negative $C_j - Z_j$ to enter the solution.
2. Determine the row to be replaced by selecting that one with the smallest (non-negative) quantity-to-pivot-column ratio.
3. Calculate the new values for the pivot row.
4. Calculate the new values for the other row(s).
5. Calculate the C_j and $C_j - Z_j$ values for this tableau. If there are any $C_j - Z_j$ values less than zero, return to Step 1.

Special Cases
Infeasibility

C_j		5	8	0	0	M	M	
	Solution Mix	X_1	X_2	S_1	S_2	A_1	A_2	Qty
5	X_1	1	0	-2	3	-1	0	200
8	X_2	0	1	1	2	-2	0	100
M	A_2	0	0	0	-1	-1	1	20
	Z_j	5	8	-2	31-M	-21-M	M	1800+20M
	C_j-Z_j	0	0	2	M-31	2M+21	0	

Special Cases
Unboundedness

C_j		6	9	0	0	
	Solution Mix	X_1	X_2	S_1	S_2	Qty
9	X_2	-1	1	2	0	30
0	S_2	-2	0	-1	1	10
	Z_j	-9	9	18	0	270
	C_j-Z_j	15	0	-18	0	

Pivot Column

Special Cases
Degeneracy

C_j		5	8	2	0	0	0	
	Solution Mix	X_1	X_2	X_3	S_1	S_2	S_3	Qty
8	X_2	1/4	1	1	-2	0	0	10
0	S_2	4	0	1/3	-1	1	0	20
0	S_3	2	0	2	2/5	0	1	10
	Z_j	2	8	8	16	0	0	80
	C_j-Z_j	3	0	6	16	0	0	

Pivot Column

Special Cases
Multiple Optima

C_j		3	2	0	0	
	Solution Mix	X_1	X_2	S_1	S_2	Qty
2	X_2	3/2	1	1	0	6
0	S_2	1	0	1/2	1	3
	Z_j	3	2	2	0	12
	C_j-Z_j	0	0	-2	0	

Sensitivity Analysis
High Note Sound Company

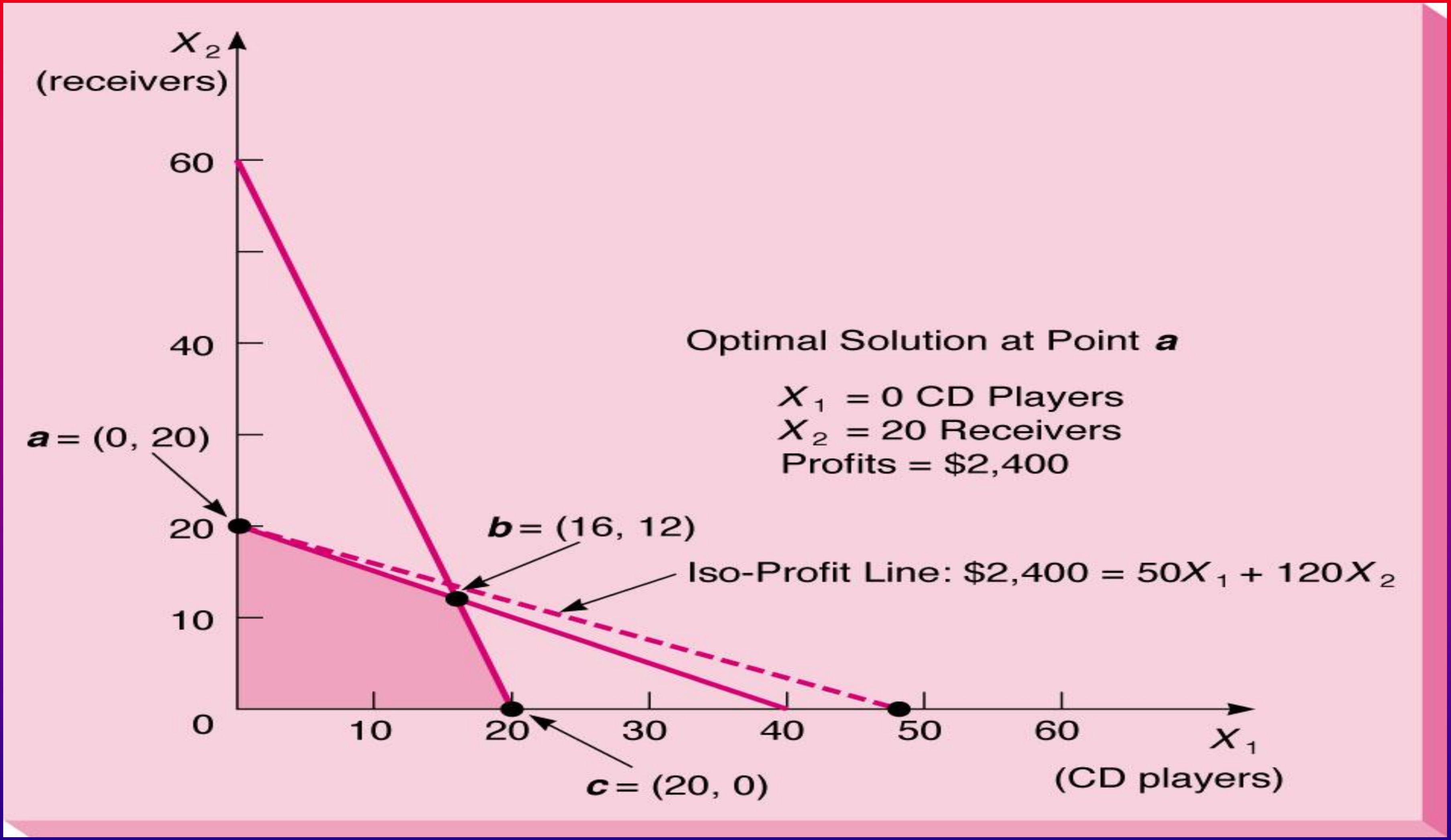
$$\text{Max}: 50X_1 + 120X_2$$

Subject to:

$$2X_1 + 4X_2 \leq 80$$

$$3X_1 + 1X_2 \leq 60$$

Sensitivity Analysis
High Note Sound Company



Simplex Solution
High Note Sound Company

C_j		50	120	0	0	
	Solution Mix	X_1	X_2	S_1	S_2	Qty
120	X_2	1/2	1	¼	0	20
0	S_2	5/2	0	-1/4	1	40
	Z_j	60	120	30	0	2400
	C_j-Z_j	0	0	-30	0	

Simplex Solution
High Note Sound Company

C_j		50	120	0	0	
	Solution Mix	X_1	X_2	S_1	S_2	Qty
120	X_2	1/2	1	¼	0	20
0	S_2	5/2	0	-1/4	1	40
	Z_j	60	120	30	0	2400
	C_j-Z_j	-10	0	-30	0	

Nonbasic Objective Function Coefficients

C_j		50	120	0	0	
	Solution Mix	X_1	X_2	S_1	S_2	Qty
120	X_2	1/2	1	¼	0	20
0	S_2	5/2	0	-1/4	1	40
	Z_j	60	120	30	0	2400
	C_j-Z_j	-10	0	-30	0	

Basic Objective Function Coefficients

C_j		50	120	0	0	
	Solution Mix	X_1	X_2	S_1	S_2	Qty
$120+\Delta$	X_2	1/2	1	¼	0	20
0	S_2	5/2	0	-1/4	1	40
	Z_j	$60+1/2\Delta$	$120+\Delta$	$30+1/4\Delta$	0	$2400+20\Delta$
	C_j-Z_j	$-10-1/2\Delta$	0	$-30-1/4\Delta$	0	

Simplex Solution
High Note Sound Company

C_j		50	120	0	0	
	Solution Mix	X_1	X_2	S_1	S_2	Qty
120	X_2	1/2	1	¼	0	20
0	S_2	5/2	0	-1/4	1	40
	Z_j	60	120	30	0	2400
	C_j-Z_j	0	0	-30	0	

Objective increases by 30 if 1 additional hour of electricians time is available.

Steps to Form the Dual

To form the Dual:

- ♣ If the primal is max., the dual is min., and vice versa.
- ♣ The right-hand-side values of the primal constraints become the objective coefficients of the dual.
- ♣ The primal objective function coefficients become the right-hand-side of the dual constraints.
- ♣ The transpose of the primal constraint coefficients become the dual constraint coefficients.
- ♣ Constraint inequality signs are reversed.

Primal & Dual

Primal:

Max : $50 X_1 + 120 X_2$

Subject to :

$$2 X_1 + 4 X_2 \leq 80$$

$$3 X_1 + 1 X_2 \leq 60$$

Dual

Min : $80 U_1 + 60 U_2$

Subject to :

$$2U_1 + 3U_2 \geq 50$$

$$4U_1 + 1U_2 \geq 120$$

Comparison of the Primal and Dual Optimal Tableaus

Primal's Optimal Solution

C_j	Solution Mix	Quantity	$50 X_1	$120 X_2	$0 S_1	$0 S_2
$7	X_2	20	1/2	1	1/4	0
$5	S_2	40	5/2	0	-1/4	1
	Z_j	$2,400	60	120	30	0
	$C_j - Z_j$		-10	0	-30	0

Dual's Optimal Solution

C_j	Solution Mix	Quantity	80 X_1	60 X_2	$0 S_1	$0 S_2	M A_1	M A_2
$7	U_1	30	1	1/4	0	-1/4	0	1/2
$5	S_1	10	0	-5/2	1	-1/2	-1	1/2
	Z_j	$2,400	80	20	0	-20	0	40
	$C_j - Z_j$		$0	40	0	20	M	M-40